MTE-11

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

PROBABILITY AND STATISTICS

(Valid from 1st January, 2022 to 31st December, 2022)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
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Maidan Garhi
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ……………………………

STUDY CENTRE: ………………………….….. **DATE:** ………………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2022**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

Course Code: MTE-11
Assignment Code: MTE-11/TMA/2022
Maximum Marks: 100

1. Which of the following statements are true? Give reasons for your answer. (10)
   a) Maximum likelihood estimators are always unique.
   b) For two events E_1 and E_2, it is known that $P(E_1) = 0$ and $P(E_2) > 0$. Then $P(E_1 / E_2) = 0$.
   c) The correlation coefficient and regression coefficients are of same sign.
   d) If X and Y are random vectors and regression of Y on X is linear, then
   $$E(Y/X) = \mu_y + \rho \frac{\sigma_y}{\sigma_x}(X - \mu_x).$$
   e) If $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(\mu, 1)$, then $S_0^2 = \sum_{i=1}^{n} (X_i - \mu)^2$ follows normal distribution.

2. a) Suppose that the following table represents the joint probability distribution of the discrete random variable (X, Y): (5)

X \ Y	1	2	3
1	0	$\frac{1}{6}$	$\frac{1}{12}$
2	$\frac{1}{9}$	0	$\frac{1}{5}$
3	$\frac{2}{15}$	$\frac{1}{4}$	$\frac{1}{18}$

   Then find
   i) $E(X)$.
   ii) Probability distribution of random variable $Z = X + Y$.
   iii) $P(Z \geq 5)$.

   b) An urn contains 2 red, 3 black and 5 white balls. If 3 ball are drawn at random without replacement, find the probabilities that (5)
   i) all 3 balls are black.
   ii) two balls are red and one ball is black.
   iii) one ball of each colour is drawn.

3. a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with p.d.f.
   $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

Find the best critical region of size α for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2, \sigma_1^2 > \sigma_0^2$. Also find its power. (5)

b) The first four moments of a distribution about the value 4 of the variable are $-1.5, 70, -30$ and 108. Find the first four moments about mean, b_1 and b_2. (5)

4. a) X is negative exponential variable having probability density function

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find mean μ and variance σ^2 of random variable X. Hence, obtain the lower limit of $[\,|X - \mu| \le 3\sigma]$ using Chebychev's inequality and compare it with its exact value. (6)

b) If X and Y are independent Poisson variates such that $P(X = 1) = 2P(X = 2)$ and $2P(Y = 2) = 3P(Y = 3)$. Find the variance of $X + 3Y$. (4)

5. a) The data for 5 years on profit earned (X) by a company and the dividend paid (Y) by it yielded the following summated values in Rs. Lac:

$\Sigma X = 100, \Sigma Y = 80, \Sigma XY = 1684, \Sigma X^2 = 2080, \Sigma Y^2 = 1398.$

Obtain

i) regression equation of Y on X.

ii) dividend (Y) for profit earned $(X) = 40$. (5)

b) Consider the following joint p.d.f.:

$$f(x, y) = \begin{cases} 1, & -y < x < y, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then

i) find $\text{Cov}(x, y)$.

ii) check whether X and Y are independent or not. (5)

6. a) Find the moment generating function of a random variable X having probability density function

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Hence obtain mean and variance of X. (5)

b) Find the mean and standard deviation for the following data: (5)

Class Interval	Frequency
0-10	5
10-20	10
20-30	24
30-40	15
40-50	6

7. a) A special die is prepared such that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are $\frac{1-K}{6}, \frac{1-2K}{6}, \frac{1-3K}{6}, \frac{1+3K}{6}, \frac{1+2K}{6}$ and $\frac{1+K}{6}$, respectively. If the die is thrown twice, find the probability of getting a sum 9. (3)

b) Let
$Y = |X+5| + |X-10| + |15-X| + |X-25| + |X-30|$.
Find X at which Y is minimum. (2)

c) Let $X_1, X_2, \ldots, X_n$ be i.i.d. Bernoulli variate with parameter p. Show that
i) $\overline{X}$ is unbiased for p.
ii) $\overline{X}$ is also UMVUE of p. (5)

8. a) For geometric distribution
$p(x) = 2^{-x}, x = 1, 2, 3, \ldots$.
Find
i) mean and variance of the distribution.
ii) $P[|X-2| \le 2]$.
iii) a lower bound for the probability computed in part (i) and compare it with the actual probability. (6)

b) The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P[x \ge 1]$ and $E = \left(\frac{X}{6} - \frac{2}{3}\right)^2$. (4)

9. a) The means $(\overline{X}, \overline{Y})$ of a bivariate frequency distribution are $(3, 4)$ and the coefficients of correlation between x, y is 0.4. If the coefficient of regression of Y on X is 1, then find two lines of regression. Also estimate X when $Y = 1$. (5)

b) In an intelligence test administered to 1000 children, the average score is 42 and standard deviation 24. Find
i) the number of children having score more than 60, and
ii) the number of children having score between 20 and 40. (5)

[You may like to use the following values:
$\phi(0.75) = 0.7734, \phi(-0.91) = 0.820, \phi(-0.083 = 0.5330, \phi(-0.09) = 0.5359,$
$\phi(0.65) = 0.7422$]

10. a) There are two bags A and B. A contains 4 white and 2 black balls and B contains 2 white and 4 black balls. One of the two bags is selected at random and two balls are drawn from it without replacement. If both the balls drawn are white, then find the probability that the bag A was used to draw the balls. (4)

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population with mean 50 and variance σ^2 (unknown). Find the estimator of σ^2 by the method of maximum likelihood. Is this estimator, obtained, unbiased? Justify your answer. (6)