MTE-08

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

DIFFERENTIAL EQUATIONS

(Valid from 1st January, 2022 to 31st December, 2022)

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term- End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……

NAME :……

ADDRESS :……

……

……

COURSE CODE: …….

COURSE TITLE : …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. DATE: ……..……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2022. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-08
Assignment Code: MTE-08/TMA/2022
Maximum Marks: 100

1. Classify the following as true or false giving reasons for your answers.

   a) The integrating factor of the differential equation

   $$xdx + ydy = m(xdy - ydx) \text{ is } \frac{1}{xy}.$$

   b) The equation $\frac{\partial^2 z}{\partial^2 x} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ is hyperbolic.

   c) The solution of the differential equation

   $$\left(D^3 + D^2 D' - DD'^2 - D'^3\right)z = 0 \text{ is}$$

   $$z = \phi_1(y + x) + \phi_2(y - x) + x^2\phi_3(y - x).$$

   d) The Pfaffian differential equation
   $a^2y^2z^2dx + b^2z^2x^2dy + c^2x^2y^2dz = 0$ is integrable.

   e) The general solution of the equation $x^2y'' + xy' - y = 0,$ defined in $[0,1]$ is given
   $y = c_1x + c_2x^{-1}.$ (10)

2. a) Find the solution of the differential equation:
   $(ax + hy + g)dx + (hx + by + f)dy = 0$ (3)

   b) Use the method of undermined coefficients to find the general solution of the differential equation:

   $$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 3e^x + 4x^2$$ (4)

   c) Solve the differential equation: $\frac{dy}{dx} - \frac{3}{2x}y = \frac{2x}{y}$ (3)

3. a) Use the method of variation of parameters to solve the differential equation:

   $$y''' + 3y' + 2y = 4e^x$$ (4)

   b) Find the integral curve of the differential equations:

   $$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$ (4)

   c) Find a particular integral of differential equation:

$\left(D^3 - D'^3\right)z = x^3y^3$ (2)

4. a) The surface temperature of a dead body changes at a rate proportional to the difference between the temperature of a body and that of the surrounding environment. The temperature of a dead body is 85° F when discovered and 74° F two hours later. If the surrounding temperature is 68° F, find the time when the body was discovered after death (take temperature of the body at the time of death as $98\cdot6^\circ$ F). (5)

b) Solve: $x^2y'' - 2xy' - 4y = x^2 + 2\ln x$ (3)

c) Given an example of an elliptic partial differential equation of 2nd order, justifying your answer. (2)

5. a) Solve the equation: $(7y - 3x + 3)dy + (3y - 7x + 7)dx = 0.$ (3)

b) Using the method of undermined coefficients, solve the equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2.$$ (3)

c) Using Charpit's method, solve the equation: $zp^2 - y^2p + y^2q = 0.$ (4)

6. a) Solve the equation: $x\frac{dy}{dx} = y + xe^{y/x}$ subject to $y(1) = 1.$ (3)

b) Solve: $(D^2 - a^2D'^2)z = x$ (2)

c) Using Charpit's method, solve: $p^2 + q^2 - 2px - 2qy + 1 = 0$ (5)

7. a) Using Lagrange's method, solve the differentiable equation:
$(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (4)

b) Solve the Laplace equation: $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$ satisfying the boundary condition:

(i) $\phi = 0$ when $y \to \infty$

(ii) $\phi = 0$ when $x = 0$

(iii) $\phi = 0$ when $x = 1$

(iv) $\phi = x(1 - x)$ when $y = 0,\ 0 < x < 1.$ (6)

8. a) Solve the Laplace equation $\frac{\partial^2u}{\partial x^2} + \frac{\partial^{2u}}{\partial y^2} = 0$ in the rectangle with
$u(0,y) = 0, u(a,y) = 0, u(x,b) = 0$ and $u(x,0) = f(x).$ (7)

b) Solve: $6y^2dx - x\left(2x^3 + y\right)dy = 0.$ (3)

9. a) Solve: $\frac{-dx}{x(x+y)} = \frac{dy}{y(x+y)} = \frac{dz}{(x-y)(2x+2y+z)}.$ (5)

b) Using the Lagrange's method, solve the differential equation
$\left(xy^3 - 2x^4\right)p + \left(2y^4 - x^3y\right)q = 9z\left(x^3 - y^3\right).$ (5)

10. a) Solve: $\frac{d}{x+y-xy^2} = \frac{dy}{x^2y-x-y} = \frac{dz}{z(y^2-x^2)}$ (4)

b) A vibrational system consisting of mass $\frac{1}{10}$kg is attached to a spring (spring constant $= 4\,\text{kg/m}$).) The mass is released from rest 1m below the equilibrium position. The motion is damped (damping constant $= 1\cdot 2$) and is being driven by an external force 5 din 4t, beginning at $t = 0$. Write the governing equations of the system and interpret the type of motion. Hence find the position of mass at time t.
(4)

c) Using $X = x - 2, Y = y + 1,$ reduce the equation $4(x-2)^2\frac{dy}{dx} = (x+y-1)^2$ to the homogeneous form of 1st order equation. (2)