MTE-01

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

CALCULUS

(Valid from 1st January, 2022 to 31st December, 2022)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2022**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

Course Code: MTE-01
Assignment Code: MTE-01/TMA/2022
Maximum Marks: 100

1. Which of the following statements are true? Justify your answers: (10)
   i) If a function f from $\mathbf{R}$ to $\mathbf{R}$ is such that $|f|$ is continuous, then f is also continuous.
   ii) The function f, defined by $f(x) = x + \sin x$, is monotonic in $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$.
   iii) $\int_0^3 |x-1|\,dx = \frac{3}{2}$.
   iv) $\frac{d}{dx}\left[\int_x^{e^x} \ell n\ t\ dt\right] = xe^x - \ell nx$.
   v) $y(x^2+4) = 2$ has oblique asymptotes.

2. a) If $y = \ell n\left[x + \sqrt{x^2+1}\right]$, check whether $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} - n^2 y_n = 0$ is true or not. (5)

   b) Find lower and upper integrals of f, defined on $[-1, 1]$, by
   $$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 2, & \text{if } x \text{ is irrational} \end{cases}.$$
   Hence, check the integrability of f on $[-1, 1]$. (5)

3. a) Check whether the function f, defined by $f(x) = \cos x - \cos 3x$, is periodic or not. (3)

   b) By dividing the internal $[0, 4]$ into 4 equal parts, find the approximate value of $\int_0^4 \frac{dx}{x+1}$, using Simpon's rule. (4)

   c) Differentiate $\sin^{-1} x$ with respect to $\cos^{-1}\left(\sqrt{1-x^2}\right)$. (3)

4. a) If, for $n \geq 1$, $I_n = \int_0^{\frac{\pi}{2}} \cos^n x \cos nx\ dx$, find the value of $2I_{n+1} - I_n$. Hence, evaluate I_2. (5)

   b) Find the derivative of $(\tan x)^{\sec x} + (\sec x)^{\cot x}$ with respect to x. (5)

5. a) Trace the curve $x^2 = y^2(x+1)^3$, stating all the properties used in the process. (10)

6. b) State Lagrange's Mean Value Theorem and use it to prove that

$1 + x < e^x < 1 + xe^x, \forall x > 0.$ (6)

b) Evaluate: $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx.$ (4)

7. a) Find the volume of the solid generated by the revolution of the curve $(a - x)y^2 = a^2 x$ about its asymptote. (6)

b) A function f is defined on $\mathbf{R}$ by

$$f(x) = \begin{cases} C^2 x, & \text{if } x \le 1 \\ 5Cx - 6, & \text{if } x > 1 \end{cases}$$

Determine the value(s) of C so that f becomes continuous on $\mathbf{R}$. (4)

8. a) Find the area of one arch of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ bounded by its base. (5)

b) Evaluate $\int \frac{(x^2 - 1)}{x^4 + x^2 + 1} dx.$ (3)

c) Find the values of $(0.98)^{5/2}$ upto 3 decimal places. (2)

9. a) Find the area included between the parabolas $y^2 = 4x$ and $x^2 = 4y.$ (5)

b) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx,$ then prove that $(n-1)(I_n + I_{n-2}) = 1.$ (5)

10. a) Differentiate $\sin^{-1} x$ with respect to $\cos^{-1} \sqrt{(1 - x^2)}.$ (2)

b) Evaluate: $\int \frac{1 + x^2}{1 + 7x^2 + x^4} dx.$ (4)

c) Find the equation of the tangent and the normal to the curve $x^2 + y^2 + 4x + 3y - 25 = 0$ at $(-3, 4).$ (4)