MTE-14

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

MATHEMATICAL MODELLING

Valid from 1st January, 2020 to 31st December, 2020

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……

NAME :……

ADDRESS :……

……

……

COURSE CODE : …….

COURSE TITLE : …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE : ……..….. DATE :…….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2020. If you have failed in this assignment or fail to submit it by December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-14
Assignment Code: MTE-14/TMA/2020
Maximum Marks: 100

1. a) Suppose that a given population can be divided into two parts: those who have a given disease and can infect others, and those who do not have it but are susceptibles. Assume that the disease spreads by contact between sick and well members of the population and the rate of spread is proportional to the number of such contacts. If the initial population of infectious individuals is 100 then
   i) Formulate a mathematical model for the given problem and write a differential equation governing it.
   ii) Find the equilibrium points of the differential equation.
   iii) Solve the given problem. What happens to the spread of the disease as (time) $t \to \infty$? (5)

   b) A particle of mass m is thrown vertically upward with velocity v_0. The air resistance is $mg\ cv^2$ where c is a constant and v is the velocity at any time t. Show that the time taken by the particle to reach the highest point is given by
   $$v_0\sqrt{c} = \tan\ (gt\sqrt{c}\,)$$ (3)

   c) The respiratory flow of air in the lungs is affected due to air pollution. If you have to model respiratory flow write four essentials for the model. (2)

2. a) A shell when projected at an angle of $\tan^{-1}\frac{1}{3}$ to the horizon falls 60 m. short of the target. When it is fixed at an angle of 45° to the horizon, it falls 80 m beyond the target. How far is the target from the point of projection? (5)

   b) Assume that a spherical raindrop evaporates at a rate proportional to its surface area. If its radius originally is 3 mm, and one-half hour later has been reduced to 2 mm, find an expression for the radius of the raindrop at any time. (3)

   c) Suppose a viscous oil, whose flow is in the laminar regime is to be pumped through a 10 cm diameter horizontal pipe over a distance of 15 km at a rate of $10^{-3}\,\mathrm{m}^3/\mathrm{s}$. Viscosity of the oil is 0.03 poise. What is the required pressure drop to maintain such a flow? (2)

3. a) If a simple pendulum of length ℓ oscillates through an angle α on either side of the mean position then find the angular velocity $\frac{d\theta}{dt}$ of the pendulum where θ is the angle which the string makes with the vertical. (3)

   b) The population $x(t)$ of a certain city satisfies the logistic law
   $$\frac{dx}{dt} = \frac{1}{100}x - \frac{1}{(10)^8}x^2$$
   where t is measured in years. Given that the population of the city is 100000 in 1980, determine the population at any time $t > 1980$. Also find the population in the year 2000. (4)

   c) Find the output yielding maximum profit for the cost function
   $$C = 0.7x^3 - 0.8x^2 + 12x + 9$$
   given that the cost price of x is ₹ 45/- per unit. (3)

4. a) Given one example each from the real world for the following, along with justification, for your example
   i) A non-linear model
   ii) A linear, deterministic model. (2)

   b) Consider arterial blood viscosity $\mu = 0.025$ poise. If the length of the artery is 1.5 cm, radius 8×10^{-3} cm. and $P = P_1 - P_2 = 4\times10^3$ dynes/cm^2 then find (i) maximum peak velocity of blood and (ii) the shear stress at the wall. (3)

   c) Suppose a planet describes an ellipse with sun S as its focus, whose major axis is $2a$ and minor axis is $2b$. Let $P(x, y)$ be the position of the planet after time t after starting from rest from perihelion position A, referred to S as origin. Let θ be the eccentric angle of the position P. Show that
   $$h\,t = ab\,(\theta - e\sin\theta)$$ (5)

5. a) Formulate a one-dimensional model describing the dynamics of phytoplankton growth $C(x, t)$ in a water mass taking into account the following: D, its diffusion coefficient, α its rate of growth, β its mortality rate due to sinking. Fixing the area of interest as $0 \le x \le 1$ and the initial concentration of phytoplankton as 20 moles / cm^3, find the concentration distribution of phytoplankton in $0 \le x \le 1$ at any time t. (8)

   b) Suppose you are driving a van down a highway. Use dimensional analysis to find the wind force you are experiencing, assuming that the force is affected by the wind density, the speed of the van and its surface area exposed to the wind direction. (2)

6. a) Consider the following system of differential equations representing a prey and predator population model
   $$\frac{dx}{dt} = x^2 - y$$
   $$\frac{dy}{dt} = x + y$$
   i) Identify all the real critical points of the system
   ii) Obtain the type and stability of these critical points. (5)

   b) A cassette player repairman finds that the time spent on his job has an exponential distribution with mean 15 min. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 18 per 9 hours a day, what is repairman's expected idle time each day? How many jobs are ahead of the set just brought in? (3)

   c) Bacterial cells grow at a rate proportional to the volume of dividing cells at that moment. If V_0 is the volume of dividing cells at any time t. Find the time at which the volume of the cells will be double its original size. (2)

7. a) Two players A and B are involved in a game. Each player has three different strategies. The pay-off table is given below:
   $$\begin{array}{c} \\ A \end{array} \begin{array}{c} B \\ \begin{bmatrix} 5 & -7 & -17 \\ 4 & 6 & -15 \\ 9 & 10 & -13 \end{bmatrix} \end{array}$$
   Find the saddle point and value of the game. (3)

b) Newton's law of cooling assumes that air at room temperature is blown past the cooling body (forced cooling). For cooling in still air (natural cooling) a better modal is to assume that the rate of temperature decrease of the cooling body is directly proportional to the $\frac{5}{4}$th power of the difference between the temperature u of the body and the temperature u_s of the surrounding air.

i) Write the law for natural cooling as a differential equation. Is this equation linear? (2)

ii) Solve the equation obtained in i) above assuming that initially, the temperature of the cooling body was u_0. (3)

c) Consider the following first-order ODE formulations

$$\frac{dn(t)}{dt} = a\{L - n(t)\},\ n(t_0) = n_0$$

Associate the physical meaning to the variables $\{t,\ n(t)\}$ and the parameters $\{a,\ L\}$ so that the above formulation becomes a mathematical model for population changes. (2)

8. a) The rate of increase of susceptible AIDS victims is proportional to the number of susceptible persons and number of infected persons. If there are S_0 susceptible persons and 1 infected person at a time to then i) set up the equation for the spread of the disease ii) solve the resulting equation iii) give a physical interpretation to the same by plotting the epidemic curve iv) write the limitations of the model. (6)

b) The sales of a company from 1993-1998 are given below:

Year	1993	1994	1995	1996	1997	1998
Sales (in lakhs of rupees)	40	45	50	55	60	65

Fit a linear curve using the least squares method. Hence find out the company's sales in 1999. (4)

9. a) Let $\rho = (w_1,\ w_2)$ be a portfolio of two securities. Find the value of w_1 and w_2 in the following situations:

i) $\rho_{12} = -1$ and ρ is risk-free.

ii) $\sigma_1 = \sigma_2$ and variance P is minimum.

iii) Variance on P is minimum and $\rho_{12} = -0.5,\ \sigma_1 = 2$ and $\sigma_2 = 3$. (6)

b) The cost of production of a substance per unit is given by the formula $C = q^2 - 4q + 1$, where q is the material cost. Find the selling price per unit, so that the profit on 100 units will be ₹ 200, if $q = 15$. Also calculate the cost of material per unit so that profit of 100 units can be maximized, if the selling price is ₹ 200. (4)

10. a) Two firms X and Y produce the same commodity. Due to production constraints, each firm is able to produce 1, 3 and 5 units. The cost of producing q_x units for firm X is ₹ $[6 + q_x^2 - 2q + 5]$ and firm Y has identical cost function ₹ $[6 + q_y^2 - 2q_y + 5]$ for producing q_y units. p is the price of one unit for firm X. We assume that the market is in equilibrium. The outcomes are the profits of the firm shown in the form of a matrix $A = \{a_{ij}\}$. Write (i) a_{11} (ii) a_{22} (iii) a_{21}, if demand function $D(p)$ is given as $D(p) = 50 - p$. (6)

b) Assume that the moon is at a distance of 300,000 kms from the earth and that it takes 28 days for it to orbit the earth once. Geostationary satellites are those which are at a rest relative to earth. Using these two statements derive the altitude of the geostationary satellite from the centre of the earth. (4)