MTE-13

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

DISCRETE MATHEMATICS

(Valid from 1st January, 2020 to 31st December, 2020)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2020**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

Course Code: MTE-13
Assignment Code: MTE-13/TMA/2020
Maximum Marks: 100

1) Which of the following statements are true and which are false? Justify your answers with a short proof or a counter-example. (20)

i) If the nonhomogeneous part of a linear nonhomogeneous recurrence relation is 2^n, then $A2^n$ is always a particular solution of the recurrence relation.

ii) If two graphs have equal number of vertices and equal number of edges, then they are isomorphic.

iii) If T is a spanning tree of K_4, then $\overline{T} = C_3$ or P_3.

iv) If a graph G is 4-colorable, then K_3 is a subgraph of G.

v) If $p \Rightarrow \sim q$ is true, then $p \wedge q$ is a contradiction.

vi) If $1, 1, 3$ are the characteristic roots of a homogenous recurrence relation, then its general solution is $a_n = (A + Bn)n + C.3^n$.

vii) For $m \geq 1$, $\sum_{\sum_{i=1}^{m} r_i = 1} \binom{1}{r_1, r_2, \ldots, r_m} = m$.

viii) $a_n = a_{\frac{n}{2}} + n, a_1 = 0$, where n is a power of 2, is a linear recurrence relation.

ix) The generating function of the sequence $\{1, 2, 3, \ldots, n, \ldots\}$ is $(1 - z)^{-2}$.

x) The statement $p \wedge \sim p$ is a contradiction.

2) a) Write down the converse of each of the following statements: (2)

i) If $n \equiv 1 \pmod 4$ for a natural number n, then $n = x^2 + y^2$ for two integers x and y.

ii) If a triangle is isosceles, two of its angles are equal.

b) Prove the following result by contradiction:

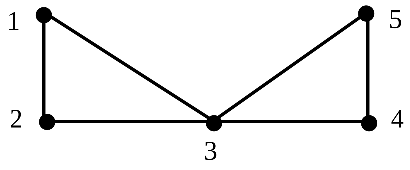
Let $f : X \to Y$ be a mapping. Suppose $f(A \cap B) = f(A) \cap f(B)$ for all subsets $A, B \subseteq X$, $f(\emptyset) = \emptyset$. Then f is a $1-1$ mapping. (4)

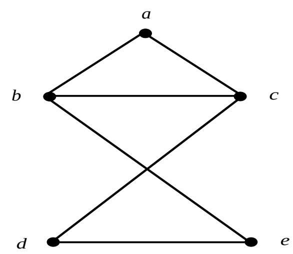
c) Give a self-conjugate partition of 21, with justification. (2)

d) Find the coefficient of x^{18} in the expansion of $(1 - x - x^2)^{10}$. (2)

3) a) Solve the recurrence relation $a_n - 4a_{n-1} + 5a_{n-2} - 2a_{n-3} = 1 + 2^n$. (4)

b) Check whether the following graphs are isomorphic or not. If they are, write the isomorphism between them. (2)





c) Show that $(p \to q) \wedge (q \to \sim p) \equiv \sim p$

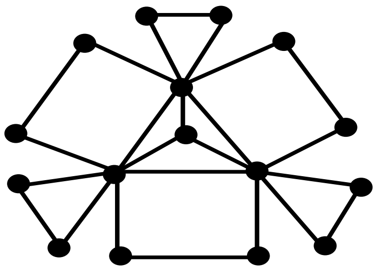
i) with truth table.

ii) without truth table. (6)

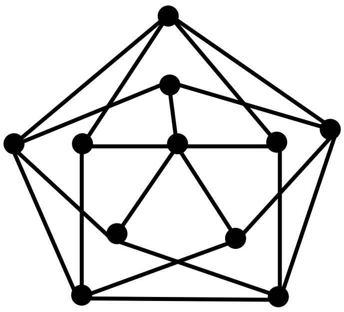
d) Prove or disprove the statement that if x and y are real numbers, then $(x^2 = y^2) \leftrightarrow (x = y)$. (3)

4. a) Prove or disprove, for two graphs G_1 and G_2, $G_1 \cong G_2 \Leftrightarrow \overline{G}_1 \cong \overline{G}_2$. (3)

b) Find a Hamiltonian cycle in the graphs given below, if it exists. If it does not exist, give reasons for its nonexistence. (4)

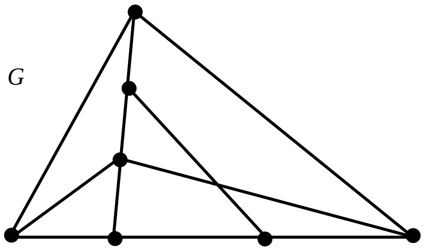


(i)



(ii)

c) Check whether the following graph G is planar or not. If it is planar give its plane drawing also. Otherwise find an edge e such that $G - e$ is planar. (3)



5. a) Prove that $S_n^{n-1} = \binom{n}{2}, \forall n \geq 2$ using i) the principle of mathematical induction, ii) combinatorial argument. (4)

b) Find the number of integer solutions of the equation

$x + y + z + w = 15,$

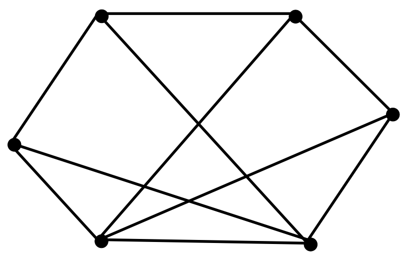
where $0 \leq x \leq 9, 1 \leq y \leq 10, 2 \leq z \leq 11, 3 \leq w \leq 12$ using

i) the principle of inclusion exclusion.

ii) generating functions. (7)

c) Find $x(G)$ and $x'(G)$ for the graph G given below: (4)

G



6. a) Show that the CNF of the Boolean expression

$X(x_1, x_2, x_3, x_4) = (x_1 \vee x_3) \wedge (x_2 \vee x_4)$ is as given below: (5)

$(x_1 \vee x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2' \vee x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3' \vee x_4) \wedge (x_1 \vee x_2 \vee x_3 \vee x_4')$

$\wedge (x_1 \vee x_2' \vee x_3' \vee x_4) \wedge (x_1 \vee x_2' \vee x_3 \vee x_4')$

b) Let p be the statement:

"If G is a k-regular graph, then G is connected."

Write $\sim p$. Which of p and $\sim p$ is true? Why? (2)

c) A five digit number is chosen at random. What is the probability that the product of its digits is 36? (3)

7) a) Solve the recurrence relation

$$a_n = a_{n-1} + n^2 + \frac{n(n+1)}{2} \qquad (5)$$

b) Consider the number of words of length n made by using letters 'a' and 'b' that do not contain 2 consecutive 'a's. Denote this number by a_n.

i) What are the values of a_1, a_2, a_3 and a_4?

ii) Derive a recurrence relation for a_n, and solve it. (5)

8) a) Determine the number of ways to seat four boys in a row of 20 chairs. (5)

b) Let $\{b_n\}_{n\geq 1}$ denote the number of bijections on a set of n elements. Find the corresponding exponential generating function. (5)