MTE-11

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

PROBABILITY AND STATISTICS

(Valid from 1st January, 2020 to 31st December, 2020)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

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THE PEOPLE'S
UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ……………………….…….. **DATE:** …………………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2020. If you have failed in this assignment or fail to submit it by December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

1. State which of the following statements are True and which are False. In case of the false statement, give the correct statement: (10)

   i) The probability of any event is always a proper fraction.

   ii) with usual notations, show that

   $$Cov(aX + bY, cX + dY) = ac\,Var(X) + bd\,Var(Y) + (ad + bc)\,Cov(X, Y)$$

   iii) In the chi-square test of goodness of fit, if the calculated value of χ^2 is zero then the fit is a bad fit.

   iv) For binomial distribution, Mean = Mode = Median.

   v) Mean lies between median and mode.

2. a) An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that: (i) two of the balls drawn are white, (ii) one is of each colour, (iii) none is red, (iv) at least one is white. (4)

   b) The first four moments of a distribution about the value 4 of the variable are $-1.5, 17, -30$ and 108. Find the moments about mean, β_1 and β_2. Find also the moments about (i) the origin, and (ii) the point $x = 2$. (6)

3. a) A random variable X has the following probability function:

   Values of X, x: 0   1   2   3   4   5   6   7

   p(x):   0   k   2k   2k   3k k^2 $2k^2$ $7k^2 + k$

   (i) Find k, (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$, (iii) If $P(X \leq a) > \dfrac{1}{2}$, Find the minimum value of a, and (iv) Determine the distribution function of X. (5)

   b) Calculate the standard deviation and mean deviation from mean if the frequency function $f(x)$ has the form: (5)

   $$f(x) = \begin{cases} \dfrac{3 + 2x}{18}, & \text{for } 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

4. a) Let X and Y be jointly distributed with p.d.f:

$$f_{XY}(x,y) = \begin{cases} \dfrac{1}{4}(1+xy), & |x|<1, |y|<1 \\ 0, & \text{otherwise} \end{cases}$$

Show that X and Y are not independent by X^2 and Y^2 are independent. (5)

b) Let X be a random variable with the following probability distribution:

x	:	-3	6	9
P(X = x)	:	1/6	1/2	1/3

Find E(X) and $E(X^2)$ and using the laws of expectation, evaluate $E(2X+1)^2$. (5)

5. a) If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|X-\mu| > 2.5\} < 0.47$, where μ is the mean of X, while the actual probability is zero. (5)

   b) The probability of a man hitting a target is $\dfrac{1}{4}$:

   i) If he fires 7 times what is the probability of his hitting the target at least twice?

   ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\dfrac{2}{3}$? (5)

6. a) If X and Y are independent Poisson variates such that

   $P(X=1) = P(X=2)$ and $P(Y=2) = P(Y=3)$.

   Find the variance of $X - 2Y$. (4)

   b) A sample of 100 items is taken at random from a batch known to contain 40% defectives. What is probability that the sample contains: (i) at least 44 defectives, (ii) exactly 44 defectives? (6)

7. a) The equations of two regression lines obtained in a correlation analysis are as follows:

   $3X + 12Y = 19, 3Y + 9X = 46$

   Obtain

   i) the value of correlation coefficient,
   ii) mean values of X and Y, and
   iii) the ratio of the coefficient of variability of X to that of Y. (5)

   b) Twenty-five pairs of value of variates X and Y led to the following results:

   $N = 25, \ \sum X = 127, \ \sum Y = 100, \sum X^2 = 760, \ \sum Y^2 = 449$ and $\sum XY = 500$

   A subsequent scrutiny showed that two pairs of values were copied down as:

X	Y
8	14
8	6

instead of

X	Y
8	12
6	8

Obtain the correct value of the correlation coefficient. (5)

8. a) The standard deviation of a population is 2.70 cms. Find the probability that is random sample of size 66 (i) the sample mean will differ from the population mean by 0.75 cm or more, and (ii) the sample mean will exceed the population mean by 0.75 cm. or more (given that the value of the standard normal probability integral from 0 to 2.25 is 0.4877). (5)

b) When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows:

No. of mistakes in a page (x) : 0 1 2 3 4 5 6

No. of pages (f) : 275 72 30 7 5 2 1

Fit a Poisson distribution to the above data and test the of goodness of fit. (5)

9. a) The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? (4)

b) i) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance.

ii) Show that the sample mean $\bar{x}$, is sufficient for estimating the parameter λ of the Poisson distribution. (6)

10. a) Let X have a p.d.f. of the form:

$$f(x, \theta) = \begin{cases} \dfrac{1}{\theta} e^{-x/\theta}, 0 < x < \infty, \theta > 0 \\ 0 \quad \text{,elsewhere.} \end{cases}$$

To test $H_0 : \theta = 2$, against $H_1 : \theta = 1$, use the random sample x_1, x_2 of size 2 and define a critical region: $W = \{(x_1, x_2) : 9.5 \le x_1 + x_2\}$.

Find
i) Power of the test.
ii) Significance level of the test. (6)

b) Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$ for the parameter θ of the distribution:

$$f(x, \theta) = \frac{1 + \theta}{(x + \theta)^2}, 1 \le x < \infty \quad (4)$$