MTE-10

# ASSIGNMENT BOOKLET

**Bachelor's Degree Programme**

**Numerical Analysis**

**(Valid from 1st January, 2020 to 31st December, 2020)**

**School of Sciences**
**Indira Gandhi National Open University**
**Maidan Garhi, New Delhi**
**(For January 2020 cycle)**

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formatting Your Assignments**

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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| | |
|---|---|
| | **ROLL NO. :**.......................... |
| | **NAME :**.......................... |
| | **ADDRESS :**.......................... |
| | .......................... |
| | .......................... |
| **COURSE CODE :** ......................... | .......................... |
| **COURSE TITLE :** ........................ | |
| **STUDY CENTRE :** ........................ | **DATE**.......................... |

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7) This assignment is valid only up to $31^{st}$ December, 2020. If you fail in this assignment or fail to submit it by $31^{st}$ December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9) **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

# Assignment

**Course Code: MTE-10**
**Assignment Code: MTE-10/TMA/2020**
**Maximum Marks: 100**

1) a) Solve the following system of linear equations using the Gauss elimination method with partial pivoting: (4)

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ 3x_1 + 2x_2 - 4x_3 &= 1 \\ x_1 + 4x_2 - 2x_3 &= 2 \end{aligned}$$

b) Using the Horner's method find the values of $f(4)$ and $f'(4)$ for the polynomial $f(x) = x^4 + 2x^3 - x^2 + 1$. (2)

c) Set up the iteration method in matrix form for the following system of linear equations:

$$\begin{aligned} 3x_1 - x_4 &= 1 \\ 4x_1 - x_2 + x_3 &= 4 \\ x_1 + 2x_3 &= 1 \\ -x_1 - x_2 - x_4 &= 2 \end{aligned}$$

Further, determine whether the method converges or not. (4)

2) a) Given the following data, estimate the value of $f(5)$ using

i) Lagrange's interpolation

ii) Newton's divided difference interpolation (6)

| $x$ | 1 | 4 | 6 | 7 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | -3 | 9 | 10 | 9 | 8 |

b) Find the inverse of the following matrix, using Gauss Jordan method. (4)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

3) a) Perform iterations of Newton-Raphson method to approximate a root of the equation $f(x) = x^4 - x^3 + x - 1 = 0$, until the roots at successive iterations are closer than $10^{-5}$. How many iterations do you need for this much accuracy? (4)

b) To approximate the value of $f'''(x_k)$, the following formulas are used

$$f_k''' = \frac{1}{h^3}[f_{k+3} - 3f_{k+2} + 3f_{k+1} - f_k]$$

$$f_k''' = \frac{1}{2h^3}[f_{k+2} - 2f_{k+1} + 2f_{k-1} - f_{k-2}]$$

Which formula do you find more accurate, and why? (6)

4) a) Compute the largest eigenvalue in magnitude, and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 3 \\ 1 & 4 & -1 \end{bmatrix}$$

by performing four iterations of the power method. (4)

b) Perform four iterations of the inverse power method to compute the smallest eigenvalue in magnitude, and the corresponding eigenvector of the matrix $A$ given in part a) above. (4)

c) Find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 4 \\ 2 & 9 & 8 \\ 6 & 5 & 2 \end{bmatrix}$ using LU decomposition method. (4)

d) Use Euler's method to solve the $I.V.P.$
$y' = \frac{x-y}{2}$, on $[0,3]$, with $y(0) = 1$, and $h = 0.5$ (3)

5) a) From the values of $f(x) = xe^x$ given in the table

| $x$ | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 |
|---|---|---|---|---|---|
| $f(x)$ | 10.8894 | 12.7032 | 14.7787 | 17.1489 | 19.8550 |

find $f''(2.0)$ using the central difference formula of $O(h^2)$ for $h = 0.1$ and $h = 0.2$. Calculate T.E. and actual error. (6)

b) Find the minimum number of intervals required to evaluate $\int_0^1 e^{-x^2+1}dx$ with an accuracy of $\frac{1}{2} \times 10^{-4}$, by using Trapezoidal rule. (5)

c) Using the classical fourth order Runge-Kutta method, find the approximate value of $y(0.6)$ for the initial value problem

$$\frac{dy}{dx} = \sin xy, \quad y(0) = 1$$

with the step size $h = 0.2$. (4)

6) a) Compute $\int_0^4 f(x)dx$ using the Romberg integral technique on the trapezoidal integrals evaluated by the trapezoidal rule taking $h = 1$ and $h = 0.5$. The tabulated values are given below. (5)

| $x$ | 0 | 0.5 | 1 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | 3 | 2 | 2.5 | 2.9 | 3.6 | 4 | 1.8 |

b) Calculate the fourth degree Taylor polynomial about $x_0 = 0.5$ for the function $f(x) = \sin^{-1}\sqrt{x}$. (5)

7) a) Consider the following data

| $x$ | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 |
|---|---|---|---|---|---|
| $f(x)$ | 0.76519 | 0.62008 | 0.45540 | 0.28181 | 0.11036 |

Use stirling's formula to approximate $f(1.5)$ with $x_0 = 1.6$ (5)

b) From the following data find $f'(5)$. (5)

| $x$ | 1 | 3 | 4 | 6 |
|---|---|---|---|---|
| $f(x)$ | 14 | 2 | 8 | 9 |

8) a) Find the interval of unit length that contains the smallest positive root of the equation $f(x) = x^3 - 5x^2 + 1 = 0$. Starting with this interval, find an interval of length 0.05 or less that contains the root, by Bisection method. (5)

b) Taking the endpoints of the last interval obtained in part a) above as the initial approximations, perform two iterations of the secant method to approximate the root. (3)

c) Determine the maximum error in quadratic interpolation at equispaced points. (2)

9) a) Find a root of the equation $3x^3 + 10x^2 + 10x + 7 = 0$ which is close to $-2.0$, using the Berge -Vieta method. Perform two iterations of the method. (5)

b) From the data

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | .5 | 1.3 | .8 | 1.5 |

interpolate the value of $f(2.8)$ using the Newton's backward difference formula. (5)