MTE-08

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

DIFFERENTIAL EQUATIONS

Valid from 1st January, 2020 to 31st December, 2020

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: …………………………….…

STUDY CENTRE : ……………………….….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2020. If you have failed in this assignment or fail to submit it by December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-08
Assignment Code: MTE-08/TMA/2020
Maximum Marks: 100

1. Classify the following statements as true or false giving reasons for your answers.

   i) General solution of the differential equations $x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$, must contain four arbitrary constants.

   ii) The set of real (or complex) solutions of equation $y'(x) + P(x)y(x) = Q(x)$, forms a real (or complex) vector space.

   iii) $\sin x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ in $]0, \pi[$ is linear homogeneous equation.

   iv) The partial differential equation

   $$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} - x^m y^n = 0$$

   is a reducible homogeneous equation.

   v) The partial differential equation $u\frac{\partial u}{\partial x} = e^y + \sin x,\ u = u(x,\ y)$, is a quasi-linear p.d.e. (10)

2. a) Using the method of undetermined coefficients, write the trial solution of the equation

   $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = xe^{-x}\cos 2x$$

   and hence solve it. (3)

   b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is $290^\circ C$ and the substance cools from $370^\circ C$ to $330^\circ C$ in 10 minutes, find when the temperature will be $295^\circ C$. (3)

   c) Solve the following differential equations

   i) $y\sin 2x\ dx = (1 + y^2 + \cos^2 x)\,dy$ (2)

   ii) $(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)\ dy = 0$ (2)

3. a) Interpret the initial value problem

   $$\frac{d^2\theta}{dt^2} + \beta^2\theta = 0,\ \theta(0) = \theta_0,\ \left.\frac{d\theta}{dt}\right)_{t=0} = \omega_0$$

   for any physical situation and hence solve the problem. (3)

   b) A series RLC circuit with $R = 6\ ohm$, $C = 0.02$ Farad and $L = 0.1$ has no applied voltage. Find the subsequent current in the circuit if the initial charge, on the capacitor is q_0 and the initial current is zero. (3)

c) Solve: $xy' - y = e^{y'}$. Also obtain its singular solution. (2)

d) Solve the differential equation $xdy - (3y + x^5 y^{1/3})\,dx = 0$. (2)

4. a) A mass weighing 39.5 kg. stretches a spring $\frac{1}{4}m$. At $t = 0$, the mass is released from a point $\frac{3}{4}m$ below the equilibrium position with an upward velocity of $\frac{5}{4}m/\sec$. Determine the function $x(t)$ that describes the subsequent free motion. (3)

b) Let y_1 and y_2 be two solutions of the equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. If $W(y_1, y_2)$ is the Wronskian of y_1 and y_2, show that

$$a_2(x)\frac{dW}{dx} + a_1(x)W = 0.$$ (2)

c) Solve the differential equation $y'' = 1 + (y')^2$. (2)

d) Solve the following differential equation by changing the independent variable

$$x^2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin x^2,\ x > 0.$$ (3)

5. Apply the method of variations of parameters to solve the following differential equations:

a) $x^2y'' + xy' - y = x^2e^x$ (3)

b) $y'' + a^2y = \operatorname{cosec} ax$ (3)

c) Solve the equation $\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - \sin^2 xy = \cos x - \cos^3 x$ by changing the independent variable. (4)

6. Find the integral curves of the following equations:

a) $$\frac{dx}{x^2 - y^2 - yz} = \frac{dy}{x^2 - y^2 - zx} = \frac{dz}{z(x-y)}.$$ (3)

b) $$\frac{dx}{x^2 + y^2} = \frac{dy}{2xy} = \frac{dz}{z(x+y)}.$$ (3)

c) Find the integral surface of the partial differential equation

$$(x-y)\,y^2p + (y-x)x^2q = (x^2 + y^2)z$$

through the curve $xz = a^2,\ y = 0$. (4)

7. a) Solve the following equations:

(i) $(D^2 + DD' + D + D' + 1)\,z = 0$. (2)

(ii) $(D^2 - 2DD' + D'^2)\,z = \tan(y + x)$. (4)

b) Solve: $z(p - q) = z^2 + (x + y^2)$. (4)

8. a) Solve the differential equation $x^3 p^2 + x^2 yp + a^3 = 0$ and also obtain its singular solution, if it exists. (3)

b) The differential equation satisfied by a beam uniformly loaded $(W \; kg / meter)$, with one end fixed and the second end subjected to a tensile force-*P*, is given by

$$EI \frac{d^2 y}{dx^2} = Py - \frac{1}{2} Wx^2,$$

where *E* is the modulus of elasticity and *I* is the moment of inertia. Show that the elastic curve for the beam with conditions $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$, is given by

$$y = \frac{W}{Pn^2}(1 - \cosh nx) + \frac{Wx^2}{2P}, \text{ where } n^2 = \left(\frac{P}{EI}\right).$$ (3)

c) For $0 < x < 5$ and $t > 0$, solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ satisfying the conditions $u(t, 0) = u(t, 5) = 0, \; u(0, x) = x$. (4)

9. a) Find $f(y)$ so that equation $f(y)dx - zx\, dy - xy \ln y\, dz = 0$ is integrable. Also obtain the corresponding integral using Natani's method. (5)

b) Find the differential equation of the family of surfaces $\phi[z(x+y)^2, \; x^2 - y^2] = 0$. (5)

10. a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, \; z = 1$. (6)

b) Show that the complete integral of $z = px + qy - 2p - 3q$ represents all possible planes through the points $(2, 3, 0)$. (2)

c) Find the values of n for which the equation $(n-1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} u_y$ is

i) parabolic ii) hyperbolic. (2)