MTE-07

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

ADVANCED CALCULUS

Valid from 1st January 2020 to 31st December 2020

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. :…………………………………………

NAME :…………………………………………

ADDRESS :…………………………………………

…………………………………………

…………………………………………

COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ……………………….…... **DATE:** …………………………………….

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December 31st, 2020. If you have failed in this assignment or fail to submit it by December 31st, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the programme guide.

We wish you good luck.

Assignment

(To be done **after** studying the course material)

Course Code: MTE-07
Assignment Code: MTE-07/TMA/2020
Maximum Marks: 100

1. Which of the following statements are true? Give reasons for your answer.
[Note: Suppose the given statements is $\lim_{(x,y)\to(0,0)} (x+y) = 2$. Simply saying that this is false is not enough. In this case you should show that the limit is zero. Similarly, if you believe that a particular statement is true, then you have to give a reason for saying so. For example, if the is like the one given below.

"A function of two variables which is continuous at a point need not have any of the partial derivatives at that point."

This statement is true. Here you can give an example of a function f which is continuous at some point and which does not have any of the partial derivatives at that point.]

a) $\lim_{x\to\infty} \cos x = 1$

b) The projection π_3 defined on a domain $\mathbf{R}^3$ is a vector-valued function.

c) Even if both the partial derivatives of $f : \mathbf{R}^2 \to \mathbf{R}$ exist at a point (a,b), the function f may still not be continuous at (a,b).

d) The function $F : \mathbf{R}^2 \to \mathbf{R}$ defined by $F(x,y) = (y+2, x+y)$ is locally invertible at any $(x,y) \in \mathbf{R}^2$.

e) The function $f(x,y,z) = e^{xyz}$ is integrable over $[0,2]\times[0,2]\times[0,2]$. (10)

2. a) Evaluate the following limits

i) $\lim_{x\to\infty} \frac{x+\sin x}{x+\cos x}$

ii) $\lim_{x\to 0} \frac{5^x - 2^x}{x}$ (4)

b) Verify Euler's relation for the function

$$f(x,y) = \frac{x^3+y^3}{x+y}$$ (3)

c) Find the slopes of the tangents at the point $(1,2,22)$ to curves of intersection of the planes $x=1$ and $y=2$ and the surface $z = 2x^2 + 5y^2$. (3)

3. a) Find $\frac{dw}{dt}$, where

i) $w = x^2 + y^2 + 2x + 3y,\ x = \cos t, y = \sin t,$ at $t = \frac{\pi}{2}$

ii) $w = xy + z, x = \cos t, y = \sin t, z = t,$ at $t = 0$. (5)

b) Check whether the following functions are differentiable at the point given against them:

i) $f(x, y) = y^3 + y \sin 2x + e^{x+y}$ at $(1, -1)$.

ii) $f(x, y) = |x = 1|$ at $(1, 0)$. (5)

4. a) Let f be a function given by

$$f(x, y) = \left(\frac{|x|}{1+|x|}, \frac{|y|}{1+|y|} \right)$$

Check whether the composite function $g \circ f$ exists, where

$g : \{(x, y) : x^2 + y^2 \le 4\} \to \mathbf{R}$ is defined by $g(x, y) = x + y$ (2)

b) Find the range of the function f defined by $f(x, y) = 10 - x^2 - y^2$ for all (x, y) for which $x^2 + y^2 \le 9$. Sketch two of its level curves. (4)

c) Check whether both the partial derivatives $f_x(0,0)$ and $f_y(0,0)$ exist for the following functions:

i) $f(x, y) = x^{1/3} y^{1/3}$

ii) $f(x, y) = \begin{cases} \frac{x}{y} + \frac{y}{x}, & (x, y) \ne (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$ (4)

5. a) Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \ne (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

i) Show that $f(0, y)$ and $f(x,0)$ are each continuous functions of one variable.

ii) Is f continuous at $(0,0)$? Give reason your answer.

iii) Is f differentiable at $(0,0)$? Give reason your answer. (6)

b) Calculate all the four second-order partial derivatives of the following functions:

i) $f(x, y) = \cos(x^2 + y^2)$

ii) $f(x, y) = \sin\left(\frac{x}{y}\right)$ (4)

6. a) Evaluate f_{xy} at a point (x, y) for the function f defined by

$$f(x, y) = x^5 + 10x^3y^3 + 8y^4$$

Verify that the function f satisfies the requirements of Schwarz's theorem and hence evaluate $f_{yx}(x, y)$. (3)

b) Verify that $f(x,t) = \exp(-k^2t)\sin(kx)$ satisfies the heat equation $\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$, where k is a constant. (3)

c) Check whether the following functions are homogeneous or not.

i) $\frac{x}{y} + \frac{3y}{2x} + \sin\sqrt{\frac{x}{y}}$

ii) $(x^4 + 4x^2 + y^2)x^2$ (4)

7. a) Locate and classify the stationary points of the function

$$f(x, y) = x^2 + y^2 - 6xy + 6x + 3y - 4.$$ (3)

b) Find the minimum value of the function

$f(x, y,) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$. (4)

c) If $u = \sin^{-1}(x^2 + y^2)^{1/5}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2}{5}\tan u.$$ (3)

8. a) Evaluate

$$\iiint_W \cos(x^2 + y^2 + z^2)^{3/2}\, dx\, dy\, dz,$$

where W is bounded by the sphere

$$x^2 + y^2 + z^2 = 25.$$ (5)

b) Check if the following integrals are independent of path and evaluate the path-independent integrals if they exists. (5)

(i) $\int_{(0,0)}^{(1,2)} (2x3^y + y)dx + (x^2e^y + x - 2y)dy$

(ii) $\int_{(0,0)}^{(2,2)} (x\sin xy + y\cos xy)dx + (x^2e^y) + x - 2y\ dy$

9. a) Find the surface area of the portion of the paraboloid $z = 25 - x^2 - y^2$, which lies above the xy-plane. (4)

b) Calculate the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ for $x = \sqrt{w}\, u \cos v, y = \sqrt{w}\, u \sin v$ and $z = w - 1$ at the point $\left(5, \frac{\pi}{2}, 3\right)$ (3)

c) Find the integral of $f(x, y) = x^4 + y^2$ over the region bounded by $y = x, y = 2x$ and $x = 2$. (3)

10. a) Use Green's theorem, and apply it to evaluate

$$\int_C (3x^2 - 4y)\,dx - (2x + y^3)\,dy$$

where C is the ellipse $4x^2 + 9y^2 = 25$. (5)

b) Find the second order Taylor polynomial of

$f(x, y) = \sin xy$ about the point $(1, \pi/2)$. (3)

c) Let the function f be defined by

$$\begin{aligned} f(x, y) &= \frac{3x^2 y^4}{x^4 + y^8}, \quad && (x, y) \neq (0, 0) \\ &= 0, && (x, y) = (0,0) \end{aligned}$$

Show that f has directional derivatives in all direction at $(0, 0)$. (2)