MTE-05

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
Analytical Geometry
(Valid from 1st January, 2020 to 31st December, 2020)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(For January 2020 cycle)

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. :..........................

NAME :..........................

ADDRESS :..........................

....

....

COURSE CODE : ....

COURSE TITLE : ....

STUDY CENTRE : .... **DATE**..........................

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7) This assignment is valid only up to 31st December, 2020. If you fail in this assignment or fail to submit it by 31st December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9) **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

Assignment

Course Code: MTE-05
Assignment Code: MTE-05/TMA/2020
Maximum Marks: 100

1) Which of the following statements are **true**, and which are **false**? Justify your answer with a short proof or a counter-example. (20)

i) The set of all the points (x,y,z) satisfying the equation $x-z=z-y$ represents a line.

ii) Any two conics can intersect at at most two points.

iii) The circle $(x-1)^2+y^2=1, z=0$ lies inside the sphere centred at the origin, and having radius $2\sqrt{2}$.

iv) If a curve C is symmetric about both the coordinate axes, then C is symmetric about the origin.

v) Every planar section of an ellipsoid is an ellipse.

vi) If α, β, γ are direction ratios of a line, then so are $\alpha^2, \beta^2, \gamma^2$.

vii) The projection of the line segment joining $(1,2,-1)$ and $(4,2,-1)$ on the line $x=y=z$ is 0.

viii) The polar equation $r=\theta$ represents a conic.

ix) If P is a point on an ellipse with a focus F, then PF is always greater than PM, where M is the foot of the perpendicular drawn from P to a directrix of the ellipse.

x) $x^2+y^2+z^2=xyz$ is the equation of a cone.

2) a) If the equation of a parabola with the focus at $(3,-4)$ and the directrix $x+y=2$ is $x^2+y^2-2xy-8x+20y+c=0$, then what is the value of c? (2)

b) Find the equation of the plane passing through the line of intersection of the planes $2x+3y+z=4$ and $x+y+z=2$, and which is perpendicular to the plane $2x+3y-z=3$. (3)

c) What is the new equation of the conic $x^2+y^2+4x-2y+3=0$, when

i) the origin is shifted at $(2,-1)$, followed by a rotation of axes through 45° ?

ii) the axes are rotated through 45°, followed by the shifting of the origin at $(2,-1)$?

Are the equations in i) and ii) above the same? Why? (4)

d) Does there pass a plane through the lines $\frac{x+4}{3}=\frac{y}{2}=\frac{z-1}{3}$ and $\frac{x}{2}=\frac{y-1}{1}=\frac{z+1}{1}$? Justify. (2)

e) Find the equation of the right circular cone whose vertex is $(1,0,1)$, the axis is $x-1=y-2=z-3$, and the semi-vertical angle is 30°. Also, find the section of the cone by the coordinate planes. (4)

3) a) A right circular cylinder passes through the point $(1,-1,4)$ and has the axis along the line $\frac{x-1}{2}=\frac{y-3}{5}=\frac{z+1}{3}$. Is this information sufficient to determine the equation of the cylinder? If it is, determine the equation of the cylinder. Otherwise, state another condition so that the equation can be determined uniquely, and also find the equation. (2)

b) Show that $x = y = z + 1$ is a secant line of the sphere $x^2 + y^2 + z^2 - x - y + z - 1 = 0$. Also find the intercept made by the sphere on the line. (2)

c) Reduce the following equations to standard form, and then identify which conicoids they represent. Further, give a rough sketch of each. (6)

i) $x^2 + y^2 + 2x - y - z + 3 = 0$

ii) $3y^2 + 3z^2 + 4x + 3y + z = 9$

4) a) Find the equation of the cone with the vertex at $(1, -1, 2)$ and the base curve as $(z+1)^2 = x + 2, y = 3$. (4)

b) Find the nature of the planar section of the conicoid $\frac{x^2}{3} - \frac{y^2}{4} = z$ by the plane $x + 2y - z = 6$ (2)

c) Trace the conicoid represented by $x^2 + 2z^2 = y$. Also describe its sections by the planes $x = c, \forall c \in \mathbb{R}$. (4)

5) a) Does there exist a plane targent to $x^2 - 2y^2 + 2z^2 = 8$ and which passes through $2x + 3y + 2z = 8, x - y + 2z = 5$? Justify your answer. (5)

b) Consider two lines L_1 and L_2 whose direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are given by the equations for l, m, n :

$$al + bm + cn = 0, fmn + gnl + hlm = 0,$$

where $abc \neq 0$. Show that if $L_1 \perp L_2$, then $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$. (5)

6) a) Give the equation of a conic which is symmetric to the line $x + 2 = 0$. (2)

b) Find the equation of the hyperbola with vertices $(1, -4)$ and $(1, 4)$, and foci at $(1, -6)$ and $(1, 6)$. (4)

c) Derive the equation (23) at page 42 of Unit 2, which represents the polar equation of a conic when the directrix L corresponding to a focus F is taken to the right of F. (4)

7) a) Check whether the points $(1, -1, -2), (1, -4, 2), (3, 0, 2), (4, -3 - 2)$ are coplanar or not. If they are coplanar, write the equation of the plane they pass through. Otherwise, change the coordinates of one of the points so that they become coplanar. In this case, find the plane passing through them. (4)

b) The normals at any point P of the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$ meet the coordinate planes in Q_1, Q_2, Q_3, respectively. Show that $PQ_1 : PQ_2 : PQ_3 :: 9 : 4 : 1$. (6)

8) a) A plane passes through (a, b, c) and cuts the axes in A, B, C, respectively, where none of these points lie on the origin O. Show that the centre of the sphere $OABC$ satisfies the equation $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. (5)

b) Find the equation of the normal to the solid $2x^2 - y^2 + 8z^2 = 11$ at a point where it intersects the line $x - 3 = z = \frac{y+1}{2}$. (5)