MTE-02

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

LINEAR ALGEBRA
(1^{st} January, 2020 to 31^{st} December, 2020)

- **It is compulsory to submit the Assignment before filling in the Term-End Examination form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives(56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from elective courses from Life Sciences, Chemistry and Physics discipline must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, at least 6 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. :..........................

NAME :..........................

ADDRESS :..........................

....

....

COURSE CODE : ....

COURSE TITLE : ....

STUDY CENTRE : .... **DATE**..........................

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) While solving problems, clearly indicate which part of which question is being solved. Your answers should be precise.

5) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Assignment responses received after the due date shall not be accepted.**

6) Please do not copy the answers from your fellow students or from the internet. Write the answers in your own language.

7) This assignment is valid only up to 31st December, 2020. If you fail in this assignment or fail to submit it by 31st December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9) **We strongly suggest that you retain a copy of your assignment response.**

We wish you good luck.

Assignment

Course Code: MTE-02
Assignment Code: MTE-02/TMA/2020
Maximum Marks: 100

1) a) Which of the following are binary operations on $\mathbf{R}$? Justify your answer.

   i) The operation $\bigtriangledown$ defined by $x \bigtriangledown y = |x| y$.

   ii) The operation $\bigtriangleup$ defined by $x \bigtriangleup y = e^{x+2y}$.

   Also, for those operations which are binary operations, check whether they are associative and commutative. (5)

   b) Find the vector equation of the plane determined by the points $(1, 0, -1)$, $(0, 1, 1)$ and $(-1, 1, 0)$. Also find the point of intersection of the line $\mathbf{r} = (1 + t)\mathbf{i} + (1 - t)\mathbf{j} + (2 + t)\mathbf{k}$ and the plane . (3)

   c) Let $\mathbf{u} = \frac{2\mathbf{i}+2\mathbf{j}+\mathbf{k}}{3}$, $\mathbf{v} = \frac{2\mathbf{i}-\mathbf{j}-2\mathbf{k}}{3}$ and $\mathbf{w} = \frac{\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{3}$. Compute the dot products $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \cdot \mathbf{w}$ and $\mathbf{v} \cdot \mathbf{w}$. Check whether $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are orthonormal. (2)

2) a) Which of the following are subspaces of $\mathbf{R}^3$? Justify your answer.

   i) $S = \{(x, y, z) \in \mathbf{R}^3 \mid x + y = z\}$

   ii) $S = \left\{(x, y, z) \in \mathbf{R}^3 \middle| 2x = 3yz\right\}$

   For those subsets which are subspaces, find a basis. (5)

   b) Check that $B = \{1, x + 1, (x + 1)^2\}$ is a basis for $\mathbf{P}_2$. Find the coordinates of $2 + x + 3x^2$ with respect to this basis. (5)

3) Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, 2x_1 + x_2 - x_3)$$

Check that T is a linear operator. Find the kernel and range of T. Find the dimension of the kernel. (10)

4) a) For the vector space $\mathbf{P}^2$, find the dual basis of $\{1 + x, 1 + 2x, 1 + x + x^2\}$? (3)

   b) Define $T : \mathbf{R}^3 \to \mathbf{R}^3$ by

$$T(x, y, x) = (-x, x - y, 3x + 2y + z).$$

   Check whether T satisfies the polynomial $(x - 1)(x + 1)^2$. (3)

   c) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear operator and suppose the matrix of the operator with respect to the ordered basis

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find the matrix of the linear transformation with respect to the basis

$$B' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$ (4)

5) a) Check whether the following system of equations has a solution. (6)

$$\begin{aligned} 3x + 2y + 6z + 4w &= 4 \\ x + 2y + 2z + w &= 5 \\ x + z + 3w &= 3 \end{aligned}$$

b) Let $T : \mathbf{P}_2 \to \mathbf{P}_2$ be defined by

$$T(a + bx + cx^2) = 2b + 3cx + (a - b)x^2.$$

Check that T is a linear transformation. Find the matrix of the transformation with respect to the ordered bases $B_1 = \{x^2, x^2 + x, x^2 + x + 1\}$ and $B_2 = \{1, x, x^2\}$. Find the kernel of T. (4)

6) a) Check whether the matrices A and B are diagonalisable. Diagonalise those matrices which are diagonalisable. (11)

i) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & -3 \\ 2 & 8 & -5 \end{bmatrix}$ ii) $B = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 2 & -1 \\ -1 & 0 & 1 \end{bmatrix}$.

b) Find inverse of the matrix B in part a) of the question by using Cayley-Hamiltion theorem. (2)

c) Find the inverse of the matrix A in part a) of the question by finding its adjoint. (3)

7) a) Let $\mathbf{P}_3$ be the inner product space of polynomials of degree at most 3 over $\mathbf{R}$ with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx.$$

Apply the Gram-Schmidt orthogonalisation process to find an orthonormal basis for the subspace of $\mathbf{P}_3$ generated by the vectors (8)

$$\{1 - x, x - x^2, x^2 - x^3\}.$$

b) Consider the linear operator $T : \mathbf{C}^3 \to \mathbf{C}^3$, defined by

$$T(z_1, z_2, z_3) = (z_1 + iz_2, iz_1 - 2z_2 + iz_3, -iz_2 + z_3).$$

i) Compute T^* and check whether T is self-adjoint.

ii) Check whether T is unitary. (6)

c) Let (x_1, x_2, x_3) and (y_1, y_2, y_3) represent the coordinates with respect to the bases $B_1 = \{(1,0,0),(1,1,0),(0,0,1)\}$, $B_2 = \{(1,0,0),(0,1,2),(0,2,1)\}$. If $Q(X) = x_1^2 - 4x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2$, find the representation of Q in terms of (y_1, y_2, y_3). (3)

d) Find the orthogonal canonical reduction of the quadratic form $-x^2 + y^2 + z^2 + xy + xz - 6yz$. Also, find its principal axes. (7)

8) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample. (10)

i) $\mathbf{R}^2$ has infinitely many non-zero, proper vector subspaces.

ii) If $T : V \to W$ is a one-one linear transformation between two finite dimensional vector spaces V and W then T is invertible.

iii) If $A^k = 0$ for a square matrix A, then all the eigenvalues of A are zero.

iv) Every unitary operator is invertible.

v) Every system of homogeneous linear equations has a non-zero solution.