MTE-01

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

CALCULUS

(Valid from 1st January, 2020 to 31st December, 2020)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: …………………………..….. **DATE:** ……………………………….…………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2020. If you have failed in this assignment or fail to submit it by December, 2020, then you need to get the assignment for the year 2021 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: MTE-01
Assignment Code: MTE-01/TMA/2020
Maximum Marks: 100

1. State whether the following statements are True or False? Justify your answers with the help of a short proof or a counter example: (10)

   a) The function f, defined by $f(x) = \cos x + \sin x$, is an odd function.

   b) $\frac{d}{dx}\left[\int_2^{e^x} \ln t \, dt\right] = x - \ln 2$.

   c) The function f, defined by $f(x) = |x-2|$, is differentiable in $[0,1]$.

   d) $y = x^2 - 3x^3$ has no points of inflection.

   e) $y = -x^2$ is increasing in $[-5,-3]$.

2. a) On which of the following intervals is $f(x) = \frac{1}{\sqrt{x-2}}$ continuous? (4)

   i) $[2,+\infty[$

   ii) $]-\infty,+\infty[$

   iii) $]2,+\infty[$

   iv) $[1,2[$

   b) Evaluate $\frac{d}{dx}[(x^2+1)^{\sin x}]$. (3)

   c) Evaluate $\int_0^{\ln 3} e^x (1+e^x)^{1/2} dx$. (3)

3. a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. (4)

   b) Let $s(t) = t^3 + 6t^2$ be the position function of a particle moving along an s-axis, where s is in metres and t is in seconds. Find the instantaneous acceleration a(t), and show the graph of acceleration with time. (3)

   c) Evaluate $\int_0^6 f(x)dx$, if $f(x)\begin{cases} x^2, & x < 2 \\ 3x-2, & x \geq 2 \end{cases}$. (3)

4. a) Let $f(x) = \tan x$.

i) Show that there exists no c in the interval $]0, \pi[$ such that $f'(c) = 0$, even though $f(0) = f(\pi) = 0$.

ii) Explain why the result in part (a) does not violate Rolle's Theorem. (4)

b) Let $f(x) = \begin{cases} 3x^2, & x \le 1 \\ ax + b, & x > 1 \end{cases}$.

Find the value of a and b so that f is differentiable at $x = 1$. (3)

c) Find the equations of the tangent lines at all inflection points of the graph of

$f(x) = x^4 - 6x^3 + 12x^3 - 8x + 3$ (3)

5. a) Prove that $\cos x \ge 1 - (x^2/2)$ for all x in the interval $[0, 2\pi]$. (3)

b) Find dy/dx if $2y^3t + t^3y = 1$ and $\dfrac{dt}{dx} = \dfrac{1}{\cos t}$. (3)

c) Find the slopes of the curve $y^2 - x + 1 = 0$ at the points $(2,-1)$ and $(2,1)$. (4)

6. a) Let $f(x) = \begin{cases} x^2 - 16x, & x < 9 \\ 12\sqrt{x}, & x \ge 9 \end{cases}$. Is f continuous at $x = 9$? Determine whether f is differentiable at $x = 9$. If so, find the value of the derivative there. (4)

b) Evaluate $\int_0^1 \tan^{-1} x \, dx$. (4)

c) Obtain the largest possible domain, and corresponding range, of the function f, defined by $f(x) = \dfrac{x-2}{3-x}$. (2)

7. Trace the curve $y = \dfrac{2x^2 - 8}{x^2 - 16}$ by showing all the properties used to trace the curve. (10)

8. a) Derive the following reduction formula:

$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx$ (5)

b) Let A denote the area of the graph of $f(x) = \sqrt{x}$ on $[0,1]$, and let B denote the area of the graph of $f(x) = x^2$ on $[0,1]$. Explain geometrically why $A + B = 1$. (5)

9. a) Find the angle between the curves $y^2 = ax$ and $ay^2 = x^3 (a > 0)$, at the points of intersection other than the origin. (6)

b) Find the intervals on which the function f defined by $f(x) = x^4 - 8x^2 + 16$ is concave upward or concave downward. (4)

10. a) Expand e^{2x} in powers of $(x-1)$, up to four terms. (4)

b) Use Simpson's method to approximate $\int_0^8 (x^2 - x + 3)\,dx$ with 8 sub-intervals. (3)

c) Find the derivative of $\ln(1 + x^2)$ w.r.t. $\tan^{-1} x$. (3)