

AOR-01

ASSIGNMENT BOOKLET

**Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)**

OPERATIONS RESEARCH

(Valid from 1st January, 2020 to 31st December, 2020)

**It is compulsory to submit the Assignment before filling in the
Term-End Examination Form.**



**School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068**

(2020)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:

NAME:

ADDRESS:

.....

.....

COURSE CODE:

COURSE TITLE:

ASSIGNMENT NO.:

STUDY CENTRE: **DATE:**

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**
We strongly suggest that you retain a copy of your answer sheets.
- 7) This assignment is valid only upto December, 2019. If you have failed in this assignment or fail to submit it by December, 2019, then you need to get the assignment for the year 2020 and submit it as per the instructions given in the programme guide.
- 8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: AOR-01
Assignment Code: AOR-01/TMA/2020
Maximum Marks: 100

1. a) A manufacturer produces two different models: X and Y, of the same product. Model X makes a contribution of Rs. 50 per unit and model Y, Rs. 30 per unit towards total profit. Raw materials r_1 and r_2 are required for production. At least 18 kg of r_1 and 12 kg of r_2 must be used daily. Also at most 34 hours of labour are to be utilized. A quantity of 2 kg of r_1 is needed for model X and 1 kg of r_1 for model Y. For each of X and Y, 1 kg of r_2 is required. It takes 3 hours to manufacture model X and 2 hours to manufacture model Y. How many units of each model should be produced to maximize the profit? (6)

- b) Obtain the dual problem of the following LP problem:

$$\text{Minimize } Z = x_1 - 3x_2 - 2x_3$$

Subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

and $x_1, x_2 \geq 0$; x_3 unrestricted in sign. (4)

2. a) Use Big-M method to solve the following LP problem.

$$\text{Minimize } Z = x_1 + 2x_2 + 3x_3 - x_4$$

Subjects to the constraints

$$x_1 + 2x_2 + 3x_3 = 15$$

$$2x_1 + x_2 + 5x_3 = 20$$

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

and $x_1, x_2, x_3 \geq 0$ (6)

- b) A pharmaceutical company is producing a single product and is selling it through five agencies situated in different cities. All of a sudden, there is a demand for the product in another five cities not having an agency of the company. The company faced the problem of deciding on how to assign the existing agencies to despatch the product to needy cities in such a way that the travelling distance is minimized. The distance between the surplus and deficit cities (in km) is given in the following table:

		Deficit cities				
		a	b	c	d	e
Surplus cities	A	160	130	115	190	200
	B	135	120	130	160	175
	C	140	110	125	170	185
	D	50	50	80	80	110
	E	55	35	80	80	105

Determine the optimum assignment schedule.

(4)

3. a) One unit of product A contributes Rs 7 and require 3 units of raw material and 2 hours of labour. One unit of product B contributes Rs 5 and requires one unit of raw material and one hour of labour. Availability of raw material at present is 48 units and there are 40 hours of labour.

- i) Formulate this problem as a linear programming problem.
 ii) Write its dual.
 iii) Solve the dual by the simplex method and find the optimal product mix and the shadow prices of the raw material and labour.

(6)

- b) A new type of water pump is to be designed for an automobile. Major specifications are given in the following table.

Activity	Predecessor Activity
A	-
B	A
C	A
D	C
E	B, C
F	D, E
G	E
H	G
I	D, F
J	I, H
K	K

Draw the network diagram for the project.

(4)

4. a) A company wants to produce three products: A, B and C. The unit profit on these products is Rs 4, Rs 6 and Rs 2, respectively. These products require two types of resources, manpower and raw material. The LP model formulated for determining the optimal product mix is as follows:

$$\text{Maximize } Z = 4x_1 + 6x_2 + 2x_3$$

Subject to the constraints

- i) Manpower constraint

$$x_1 + x_2 + x_3 \leq 3$$

- ii) Raw material constraint

$$x_1 + 4x_2 + 7x_3 \leq 9$$

and $x_1, x_2, x_3 \geq 0$

where x_1, x_2 and x_3 = number of units of product A, B and C, respectively to be produced.

- i) Find the optimal product mix and the corresponding profit of the company.
- ii) Find the range of the profit contribution of product C (i.e. coefficient c_3 of variable x_3) in the objective function such that current optimal product mix remains unchanged.
- iii) What shall be the new optimal product mix when profit per unit from product C is increased from Rs 2 to Rs 10? (5)

- b) A company has factories at F_1, F_2 and which supply to warehouse at W_1, W_2 and W_3 . Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirement are 180, 120 and 150 units, respectively. Unit shipping costs (in rupees) are as follows:

		Warehouse			Supply
		W_1	W_2	W_3	
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	450

Determine the optimal distribution for this company to minimize total shipping cost. (5)

5. a) Solve the following integer linear programming problem using the cutting plane algorithm.

$$\text{Maximize } Z = 2x_1 + 20x_2 - 10x_3$$

Subject to the constraints

$$2x_1 + 20x_2 + 4x_3 \leq 15$$

$$6x_1 + 20x_2 + 4x_3 = 20$$

and $x_1, x_2, x_3 \geq 0$. (6)

- b) A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers. (4)

Activity (i - j)	Estimated Duration (weeks)		
	Optimistic	Most Likely	Pessimistic
1 - 2	1	1	7
1 - 3	1	4	7
1 - 4	2	2	8
2 - 5	1	1	1
3 - 5	2	5	14
4 - 6	2	5	8
5 - 6	3	6	15

- i) Draw the project network.

- ii) Find the expected duration and variance for each activity. What is the expected project length?
 - iii) Calculate the variance and standard deviation of the project length.
6. a) The products of three plants X, Y and Z are to be transported to four warehouses I, II, III and IV. The cost of transportation of each unit from plants to the warehouses along with the normal capacities of plants and warehouses are indicated below:

		Warehouse				Available
		I	II	III	IV	
Plant	X	25	17	25	14	300
	Y	15	10	18	24	500
	Z	16	20	8	13	600
Required		300	300	500	500	

- i) Solve the problem for minimum cost of transportation. Are there any alternative solutions? If any, explain the methodology.
 - ii) Overtime can be used in each plant to raise the capacity by 50 per cent of the normal but the corresponding cost of trans-shipment will also increase by 10, 15 and 20 to the unit costs of production at each plant. (5)
- b) Each units of an item costs a company Rs. 40. Annual holding costs are 18 per cent of unit costs for interest charges, 1 per cent for insurance, 2 per cent allowances for obsolescence, Rs 2 for building overheads, Rs 1.50 for damage and loss, and Rs 4 miscellaneous costs. Annual demand for the item is constant at 1,000 units and each order costs Rs 100 to place.
- i) Calculate EOQ and the total costs associated with shocking the item.
 - ii) If the supplier of the item will only deliver batches of 250 units, how are the stock holding costs affected? (5)
7. a) Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 3 minutes.
- i) What is the probability that a person arriving at the booth will have to wait?
 - ii) The telephone department will install a second booth when convinced that an arrival would expect waiting for at least 3 minutes for a phone call. By how much should the flow of arrivals increase in order to justify a second booth?
 - iii) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call? (5)
- b) A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that when he starts production run, he can produce 25,000 bearings per day. The cost of holding a bearing in stock for a year is Rs 2 and the set-up cost of production run is Rs 180. How frequently should production run be made? (5)

8. a) A super market has two sales girls at the sales counters. If the service time for each customer is exponential with a mean of 4 minutes, and if possible arrive in a Poisson fashion at the rate of 10 an hour, then calculate the
- Probability of having to wait for service?
 - Expected percentage of idle time for each sales girl?
 - If a customer has to wait, what is the expected length of his waiting time? (5)
- b) A vessel is to be loaded with stock of 3 items. Each unit of item i has to weight ω_i and value π_i . The maximum cargo weight the vessel can take is 5 and the details of the three items are as follows:

i	ω_i	π_i
1	1	30
2	3	80
3	2	65

Develop the recursive equation for the above case and find the optimum cargo load without exceeding the maximum cargo weight by using dynamic programming. (5)

9. a) A book binder has one printing press, one binding machine and the manuscripts of six different books. The time required to perform the printing and binding operations for each book is given ahead. Determine the order in which books should be processed, in order to minimize the total time. Also, find elapsed time and idle time of each machine: (5)

Book	Printing time (hours)	Binding Time (hours)
1	30	80
2	120	100
3	50	90
4	20	60
5	90	30
6	100	10

- b) A marketing company has divided its area of operation into four zones (A, B, C and D). There are four sales teams available. Based on previous experience the company has estimated the annual sales if different teams are assigned to different zones as given in the table below. Find the optimal allocation of sales teams to different regions so that the total annual sales is maximized: (5)

Sales teams	Zones			
	A	B	C	D
1	5	11	8	9
2	5	7	9	7
3	7	8	9	9
4	6	8	11	12

10. Which of the following statements are true and which are false? Give a short proof of a counter example on support of your answer: (10)
- i) If a primal LP has an equality constraint, then corresponding variable in dual is unrestricted.
 - ii) In any queuing model, the expected length of the system is equal to the expected length of queue added to number of servers.
 - iii) In an EOQ model, optimal quantity increases as holding cost increases.
 - iv) Dynamic programming approach divides the given problem into a sequence of smaller sub-problems called the stages.
 - v) In the solution of a balanced transportation problem with 3 sources and 4 destinations, if the number of allocated cells is 7, then the solution is degenerate.