MMTE-005

ASSIGNMENT BOOKLET

M.Sc. (Mathematics with Applications in Computer Science)

CODING THEORY

(Valid from January 1st, 2025 – December 31st, 2025)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi 110068
2025

Dear Student,
Please read the section on assignments in the Programme Guide that we sent you after your enrolment. As you may know already from the programme guide, the continuous evaluation component has 20% weightage. This assignment is for the continuous evaluation component of the course.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : . . . .

NAME : . . . .

ADDRESS : . . . .

. . . .

. . . .

COURSE CODE : . . . .

COURSE TITLE : . . . .

STUDY CENTRE : . . . .   **DATE** . . . .

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) Write all the answers in your own words. Do not copy from the internet, from your fellow students or from any other source. If your assignment is found to be copied, it will be rejected.

7) This assignment is valid only up to December 31^{st}, 2025. If you fail in this assignment or fail to submit it by 31^{st} December 2025, then you need to get the assignment for 2026 and submit it as per the instructions given in the Programme Guide.

8) At the time of filling in the online Term End Examination form, you are supposed to give an undertaking that you have submitted/you will submit the assignment. Please ensure that you comply with the undertaking.

9) For any doubts, clarifications and corrections, write to `svenkat@ignou.ac.in`.

We **strongly** suggest that you retain a copy of your answer sheets.

Wish you good luck.

Assignment

Course Code: MMTE-005
Assignment Code: MMTE-005/TMA/2025
Maximum Marks: 100

Note: In this assignment, the notations, symbols, definitions and conventions will be as in the prescribed book 'Fundamentals of Error-Correcting Codes' by Huffman and Pless. Also, 'the book' will always mean the prescribed book.

1) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample. (10)

   i) If the weight of each element in the generating matrix of a linear code is at least r, the mininum distance of the code is at least r.

   ii) There is no linear self orthogonal code of odd length.

   iii) There is no 3-cyclotomic coset modulo 121 of size 25.

   iv) There is no duadic code of length 15 over $\mathbf{F}_2$.

   v) There is no LDPC code with parameters $n = 16$, $c = 3$ and $r = 5$.

2) a) Which of the following binary codes are linear?

   i) $\mathscr{C} = \{(0,0,0,0),(1,0,1,0),(0,1,1,0),(1,1,1,0)\}$

   ii) $\mathscr{C} = \{(0,0,0),(1,1,0),(1,0,1),(0,1,1)\}$

   Justify your answer. (3)

   b) Find the minimum distance for each of the codes. (4)

   c) For each of the linear codes, find the degree, a generator matrix and a parity check matrix. (3)

3) Let $\mathscr{C}_1$ and $\mathscr{C}_2$ be two binary codes with generator matrices

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \ G_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix},$$

respectively.

   a) Find the minimum distance of both the codes.

   b) Find the generator matrix of the code

$$\mathscr{C} = \{(\mathbf{u}|\mathbf{u}+\mathbf{v})|\mathbf{u} \in \mathscr{C}_1, \mathbf{v} \in \mathscr{C}_2\}$$

   obtained from $\mathscr{C}_1$ and $\mathscr{C}_2$ by $(\mathbf{u}|\mathbf{u}+\mathbf{v})$ construction. Also, find the minimum distance of $\mathscr{C}$. (5)

4) a) If $\mathbf{x}$, $\mathbf{y} \in \mathbf{F}_2^n$, show that

$$wt(\mathbf{x}+\mathbf{y}) = wt(\mathbf{x}) + wt(\mathbf{y}) - 2wt(\mathbf{x} \cap \mathbf{y})$$

   where $\mathbf{x} \cap \mathbf{y}$ is the vector in $\mathbf{F}_2^n$ which has 1s precisely at those positions where $\mathbf{x}$ and $\mathbf{y}$ have 1s. (2)

   (**Hint:** Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$. Suppose

$$n_1 = |\{i \mid x_i = y_i = 1\}|, n_2 = |\{i \mid x_i = 1, y_i = 0\}|, n_3 = |\{i \mid x_i = 0, y_i = 1\}|$$

   Observe that $wt(\mathbf{x}) = n_1 + n_2$ and $wt(\mathbf{y}) = n_1 + n_3$.)

b) Let $\mathscr{C}$ be a binary code with a generator matrix each of whose rows has even weight. Show that, every codeword of $\mathscr{C}$ has even weight. (3)
(**Hint:** Why is it enough to prove that sum of vectors of even weight in $\mathbf{F}_2^n$ is a vector of even weight?)

c) Show that, if $\mathbf{x} \in \mathbf{F}_3^n$,

$$wt(\mathbf{x}) \equiv \mathbf{x} \cdot \mathbf{x} \pmod 3$$

Deduce that, if $\mathscr{C}$ is a ternary self orthogonal code, the weight of each codeword is divisible by 3. (3)
(**Hint:** Observe that $x^2 = 1$ for all $x \neq 0 \in \mathbf{F}_3$)

d) The aim of this exercise is to show that every binary repetition code of odd length is perfect.

i) Find the value of t and d for a perfect code of length $2m+1$, $m \in \mathbf{N}$. (2)

ii) Show that (3)

$$\sum_{i=0}^{m} \binom{2m+1}{i} = 2^{2m}$$

(**Hint:** Start with the relation

$$2^{2m+1} = \sum_{i=0}^{2m+1} \binom{2m+1}{i})$$

iii) Deduce that every repetiition code of odd length is perfect. (2)

5) Let α be a root of $x^2+1=0$ in $\mathbf{F}_9$.

a) Check whether α is a primitive element of $\mathbf{F}_9$. If it is not a primitive element in $\mathbf{F}_9$ find a primitive element γ in $\mathbf{F}_9$ in terms of α. (4)

b) Make a table similiar to Table 5.1 on page 184 for $\mathbf{F}_9$ with the primitive element γ (3)

c) Factorise x^8-1 over $\mathbf{F}_3$. (6)

d) Find all the possible generator polynomials of a $[8,6]$ cyclic code. (2)

6) a) Let $\mathscr{C}_1$ and $\mathscr{C}_2$ be cyclic codes over $\mathbf{F}_q$ with generator polynomials $g_1(x)$ and $g_2(x)$, respectively. Prove that $\mathscr{C}_1 \subseteq \mathscr{C}_2$ if and only if $g_2(x) \mid g_1(x)$. (3)

b) Over $\mathbf{F}_2$, $(1+x) \mid (x^n-1)$. Let $\mathscr{C}$ be the binary cyclic code $(1+x)$ of length n. Let $\mathscr{C}_1$ be any binary cyclic code of length n with generator polynomial $g_1(x)$.

i) What is the dimension of $\mathscr{C}$? (1)

ii) Let w be subspace of $\mathbf{F}_2^n$ containing all the vectors of even weight. Prove that W has dimension $n-1$. (**Hint:** Consider the map $w \colon \mathbf{F}_2^n \to \mathbf{F}_2$ given by $w((a_1,a_2,\ldots,a_n)) = a_1+a_2+\cdots+a_n$.) (4)

iii) Prove that $\mathscr{C}$ is the vector space of all vectors in $\mathbf{F}_2^n$ with even weight. (4)

iv) If $\mathscr{C}_1$ has only even weight codewords, what is the relationship between $(1+x)$ and $g_1(x)$? (1)

v) If $\mathscr{C}_1$ has some odd weight codewords, what is the relationship between $1+x$ and $g_1(x)$? (2)

7) a) Let $\mathscr{C}$ be the ternary $[8,3]$ narrow-sense BCH code of designed distance $\delta = 5$, which has defining set $T = \{1,2,3,4,6\}$. Use the primitive root 8th root of unity you chose in 4a) to avoid recomputing the the table of powers. If

$$g(x) = x^5 - x^4 + x^3 + x^2 - 1$$

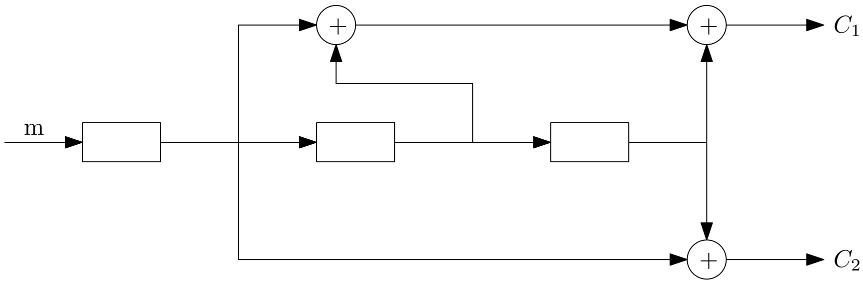

Figure 1: Encoder for convolutional code.

is the generator polynomial of $\mathscr{C}$ and

$$y(x) = x^7 - x^6 - x^4 - x^3$$

is the received word, find the transmitted codeword. (7)

b) Let $\mathscr{C}$ be the [5, 2] ternary code generated by

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Find the weight enumerator $W_{\mathscr{C}}(x,y)$ of $\mathscr{C}$. (3)

c) Find the generating idempotents of duadic codes of length $n = 23$ over $\mathbf{F}_3$. (7)
(**Hint:** Mimic example 6.1.7.)

8) a) Let

$$C = \{0000, 1113, 2222, 3331, 0202, 1313, 2020, 3131, 0022, 1131, 2200, 3313, 0220, 1333, 2002, 3111\}$$

be the Z_4-linear code. Find the Gray image of C. (4)

b) Draw the Tanner graph of the code $\mathscr{C}$ with parity check matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$ (4)

c) Find the convolutional code for the message 11011. The convolutional encoder is given in Fig. 1. (5)