MMTE-001

ASSIGNMENT BOOKLET

M.Sc. Mathematics with Applications in Computer Science (MSCMACS)

GRAPH THEORY

(1st January, 2025 to 31st December, 2025)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2025)

Dear Student,
Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1. On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : .....
NAME : .....
ADDRESS : .....
.....
.....
COURSE CODE : ....
COURSE TITLE : .....
STUDY CENTRE : ..... **DATE** .....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2. Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3. Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4. Your answers should be precise.

5. While solving problems, clearly indicate which part of which question is being solved.

6. This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7. This assignment is valid only up to 31st December, 2025. If you fail in this assignment or fail to submit it by the due date, then you need to get the assignment for the year 2026 and submit it as per the instructions given in the Programme Guide.

8. **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9. **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

Assignment

(To be done after reading the course material)

Course Code: MMTE-001
Assignment Code: MMTE-001/TMA/2025
Maximum Marks: 100

1. State whether the following statements are **true** or **false**. Justify your answers with a short proof or a counterexample (20)

   i) There exists an 8-vertex graph with three vertices of degree 3, four vertices of degree 2 and one vertex of degree 1.

   ii) The neighbour of every leaf is a cut-vertex.

   iii) Every line graph of a bipartite graph is 2-colourable.

   iv) If $(d_1, d_2, ..., d_n)$ is a graphic sequence then so is $(d_1 + 1, d_2 + 1, ..., d_n + 1)$.

   v) $\beta(K_3 \times P_3) = 4$.

   vi) A Hamiltonian graph has no cut-vertices.

   vii) The Petersen graph is 3-critical.

   viii) An n-vertex star has no perfect matching for $n \geq 3$.

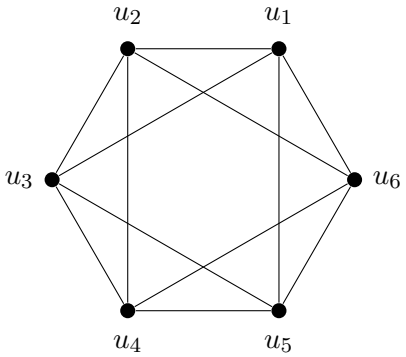
   ix) The crossing number of $K_{3,3}$ is 2.

   x) If f and g are two flows on a network N, then $\max\{f, g\}$ is also a flow.

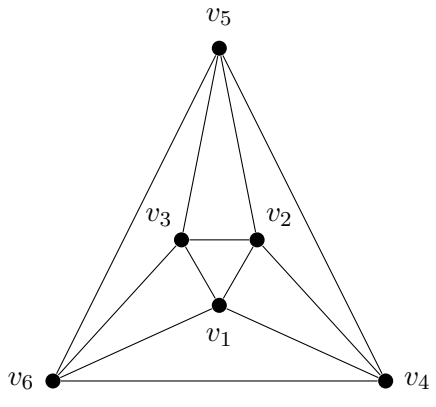
2. (a) If every cycle in a graph is even, then prove that the graph is bipartite. Is its converse true. Prove or disprove. (4)

   (b) For each n-vertex h-level complete binary tree, prove that $\log_2(n+1) - 1 \leq h \leq \frac{n-1}{2}$. (3)

   (c) Check whether the following graphs G and H are isomorphic or not. (3)



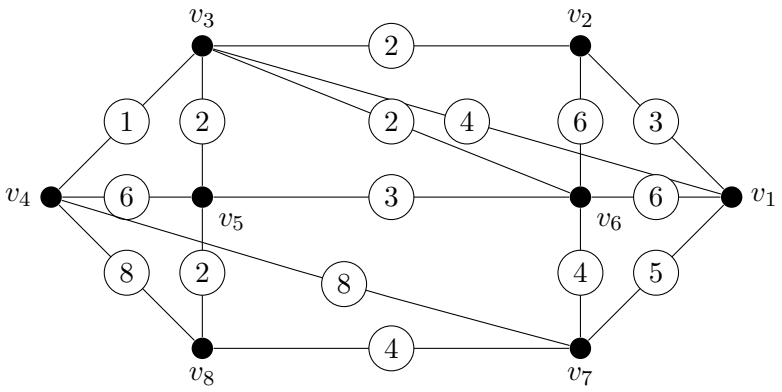
G



H

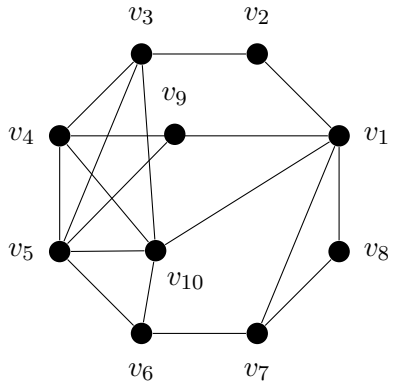
3. (a) Prove or disprove: A connected graph with order and size equal must contain exactly one cycle. (4)

   (b) Find a minimum-weight spanning tree in the following graph. (4)

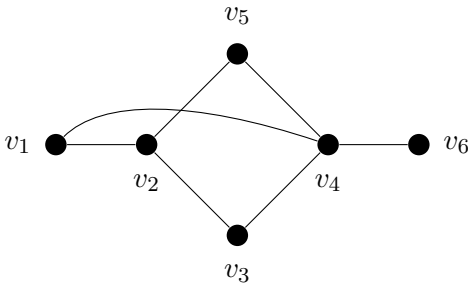


(c) Determine the number of non-planar graphs with 6 vertices. (3)

(d) Find the chromatic and edge-chromatic numbers of the following graph. (4)



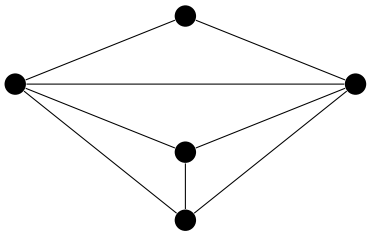
4. (a) Show that there are 14 spanning trees of the following graph. Draw all the spanning trees. (5)



(b) Is it possible that a graph is 3-chromatic but not 3-critical? If so, explain it with an example. (2)

(c) Check the sequence $(6, 5, 4, 4, 3, 1, 1, 1, 1)$ is graphic or not. Also, find a graph realising it. (3)

5. (a) Verify Euler's formula for the following plane graph. (2)

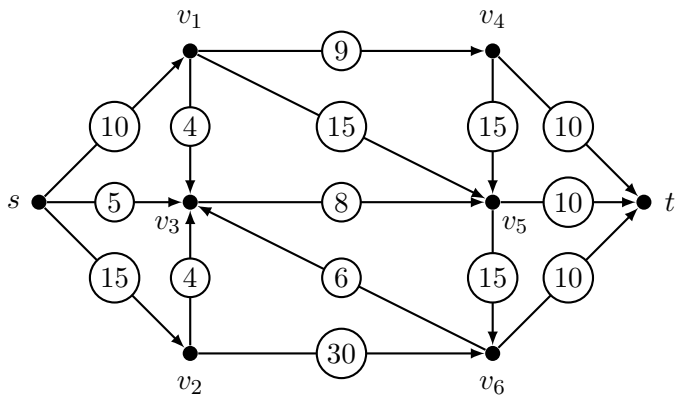


(b) Check whether the graph $C_5 \times K_5$ is planar or not. (2)

(c) For every graph G , $\kappa'(G) = \kappa(L(G))$. True or false? Justify. (3)

(d) Find the matching number of the line graph of the graph given in part (a). (3)

6. (a) What is the maximum possible flow that can pass through the following network N? Define such a flow. (4)



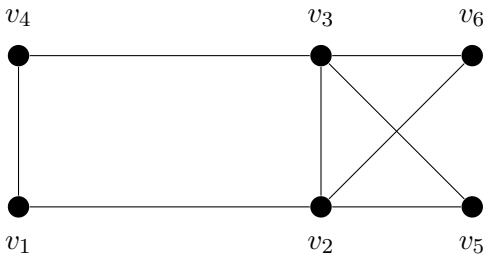
(b) Show that $[S, T]$ is an (s, t)-cut in network N give in part(a),where $S = \{s, v_1, v_2, v_3\}$ and $T = \{t, v_4, v_5, v_6\}$. Also find $Cap(S, T)$. Does N have any other (s, t)-cut with capacity smaller than $Cap(S, T)$? What is the maximum possible value of a flow in N? (4)

(c) State and prove Hall's Theorem. (4)

(d) Provide an example of a 3-regular planar graph with 8-vertices. Is this graph a maximal planar graph? Why? (3)

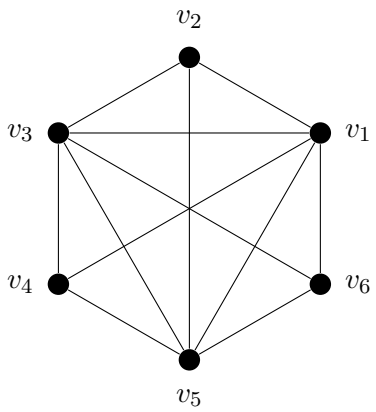
7. (a) Find the values of n and m for which the star graph $S_{n,m}$ is Eulerian. (2)

(b) Using Fleury's algorithm, find an Eulerian circuit in the following graph. (4)



(c) Prove or disprove: If G is a graph with $\chi(G)$ denoting its chromatic number, then $\kappa(G) \geq \chi(G) - 1$. (4)

8. (a) Find the line graph of the following graph? Write number of vertices and edges in the line graph. (4)



(b) Find the thickness and crossing number of the graph G given in Q.2(c)? (3)

(c) Draw $C_4 \times S_4$ with explanation. (3)