MMT-007

ASSIGNMENT BOOKLET

(Valid from 1st January, 2025 to 31st December, 2025)

M.Sc. (Mathematics with Applications in Computer Science)
Differential Equations and Numerical Solutions (MMT-007)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi,
New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. …………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MMT-007
Assignment Code: MMT-007/TMA/2025
Maximum Marks: 100

1. a) Solve the differential equation:

$$x^2 y'' + 6xy' + (6 + x^2)y = 0$$

in series about $x = 0$. (6)

b) Express $f(x) = x^4 + 3x^3 + 4x^2 - x + 2$ in terms of Legendre polynomials. (4)

2. a) Using method of Ferobenius, find the solution of the differential equation: (6)

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

near $x = 0$.

b) Find: (2)

$$L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$$

c) Find $L\{F(t)\}$, if: (2)

$$F(t) = \begin{cases} \sin\left(t - \frac{\pi}{4}\right), & t > \frac{\pi}{4} \\ 0 & , \ t < \frac{\pi}{4} \end{cases}$$

3. a) Find the Fourier transform of e^{-9x^2}. (4)

b) Find the solution of the heat conduction equation subject to the given initial and boundary conditions: (6)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1$$
$$u(x,0) = \sin(\pi x) \text{ for } 0 \le x \le 1$$
$$u(0,t) = 0 = u(1,t)$$

Using Laasonen method with $\lambda = \frac{1}{6}$ and $h = \frac{1}{3}$. Integrate for two levels.

4. a) Find the solution of the initial boundary value problem: (5)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to given initial and boundary conditions:

$$u(x,0) = 2x \text{ for } x \in \left[0, \frac{1}{2}\right]$$

and $2(1-x)$ for $\left[\frac{1}{2}, 1\right]$

$$u(0,t) = 0 = u(1,t).$$

You may use step length along x-axis, $h = 0.2$ and solve by Schmidt method with mesh ratio $\lambda = \frac{1}{6}$.

b) Show that the method. (5)

$$y_{i+1} = \frac{4}{3}y_i - \frac{1}{3}y_{i-1} + \frac{2h}{3}y_{i+1}$$

is A-stable when applied to test equation $y' = \lambda y, \lambda < 0$.

5. a) Use Fourier transforms to solve the boundary value problem: (5)

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty,\ t > 0$$

subject to the conditions:

i) $u, \frac{\partial u}{\partial x} \to 0$ as $x \to \pm\infty$

ii) $u(x,0) = f(x)$

b) Solve the initial value problem $y' = x^2 + y^2,\ y(0) = 1$, upto $x = 0.2$ using third order Taylor series method with $h = 0.1$. (5)

6. a) Using Laplace transform, solve the equation: (5)

$$\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$$

given that:

$$u(0,t) = u(2,t) = 0, u_t(x,0) = 0$$

and $u(x,0) = 10\sin 2\pi x - 20\sin 5\pi x$.

b) Using second order finite difference method, solve the boundary value problem: (5)

$$\frac{d^2y}{dx^2} = \frac{3}{2}y^2 \text{ with } y(0) = 4, y(1) = 1$$

Using step length $h = \frac{1}{3}$.

7. Find the solution of $\nabla^2 u = 0$ in R subject to the boundary conditions: (10)

$$u(x,y) = x^2 - y^2 \text{ on } x = 0, y = 0, y = 1;$$
$$u + \frac{\partial u}{\partial x} = x^2 + 2x - y^2 \text{ on } x = 1,$$

where R is the square $0 \le x \le 1, 0 \le y \le 1$, using the five point formula. Use central difference approximation in the boundary condition. Assume uniform step length $h = 1/2$ along the axes.

8. a) Use finite Fourier transform to solve: (6)

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, 0 < x < 4, t > 0$$

Subject to the conditions:

$$u(x,0) = 2x, \quad 0 < x < 4$$

and $u(0,t) = u(4,t) = 0$

b) Solve the boundary value problem: (4)

$$y'' + y + f(x) = 0$$
$$y'(0) = 0, y(1) = 0$$

by determining the appropriate Green's function by using the method of variation of parameters and expressing the solution as a definite intergral.

9. Solve the boundary value problem: (10)

$$y'' - 3y' + 2y = 2$$

with

$$y(0) - y'(0) = -1$$
$$y(1) + y'(1) = 1$$

using the second order finite difference method with step length $h = \frac{1}{2}$.

10. a) Using the generating function $J_n(x)$, prove that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$, for integer values of n. (5)

b) Evaluate: (5)

$$\int_{-1}^{1} \frac{P_n(x)}{\sqrt{1-2xt+t^2}} dx,$$

where $P_n(x)$ is Legendre polynomial.