## ASSIGNMENT BOOKLET

(Valid from $1^{\text {st }}$ January, 2024 to $31^{\text {st }}$ December, 2024)

## M.Sc. (Mathematics with Applications in Computer Science) <br> Pattern Recognition and Image Processing



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## Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, which would consist of one tutor-marked assignment. The assignment is in this booklet.

## Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO $\qquad$
NAME $\qquad$
ADDRESS $\qquad$

COURSE CODE:
COURSE TITLE $\qquad$
ASSIGNMENT NO.
STUDY CENTRE:

## DATE:

## PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programmecentre. Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto31 ${ }^{\text {st }}$ December, 2024. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by $31^{\text {st }}$ December, 2024, then you need to get the assignment for the session 2025 and submit it as per the instructions given in the programme guide.

We wish you good luck.

## Assignment

1. a) An automobile manufacturer is automating the placement of certain components on the bumpers of a limited-edition line of sports cars. The components are colour coordinated, so the robots need to know the colour of each car in order to select the appropriate bumper component. Models come in only four colours: blue, green, red, and white. Find a solution based on imaging and determine the colour of each car, keeping in mind that cost is the most important consideration.
b) Consider the two image subsets, $S_{1}$ and $S_{2}$, shown in the following figure. For $\mathrm{V}=\{1\}$, determine whether these two subsets are (i) 4 -adjacent, (ii) 8 -adjacent, or (iii) m-adjacent.

|  | $\mathrm{S}_{1}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |  |  | 0 |  | 0 |  | 1 | 0 |
| 1 | 0 | 0 | $1$ |  |  | 0 |  | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 |  |  | 1 |  | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |  |  | 0 |  | 0 |  | 0 | 0 |
| $0$ | 0 |  |  |  |  | 0 |  | 0 |  |  |  |

2. a) Two images, $f(x, y)$ and $g(x, y)$, have histograms $h_{f}$ and $h_{g}$. Give the condition under which you can determine the histograms of
i) $f(x, y)+g(x, y)$
ii) $f(x, y)-g(x, y)$
iii) $f(x, y) \times g(x, y)$
iv) $f(x, y) \div g(x, y)$
b) Write an expression for 2-D continuous convolution.
3. a) Prove that both 2-D continuous and discrete Fourier transforms are linear operations.
b) Consider a $3 \times 3$ spatial mask that averages the four closet neighbours of a point ( $\mathrm{x}, \mathrm{y}$ ), but excludes the point itself from the average.
i) Find the equivalent filter, $\mathrm{H}(\mathrm{u}, \mathrm{v})$, in the frequency domain.
ii) Show that your result is a lowpass filter.
4. The white bars in the test pattern shown are 7 pixels wide and 210 pixels high. The separation between bars is 17 pixels. What would this image look like after application of
i) A $3 \times 3$ arithmetic mean filter?
ii) A $7 \times 7$ arithmetic mean filter?
iii) A $9 \times 9$ arithmetic mean filter?

5. a) Consider an 8 -pixel line of intensity data, $\{108,139,135,244,172,173,56,99\}$. If it is uniformly quantized with 4-bit accuracy, compute the rms error and rms signal-tonoise ratios for the quantized data.
b) Prove that, for a zero-memory source with q symbols, the maximum value of the entropy is $\log \mathrm{q}$, which is achieved if and only if all source symbols are equiprobable. [Hint: Consider the quantity $\log \mathrm{q}-\mathrm{H}(\mathrm{z})$ and note the inequality In $\mathrm{x} \leq \mathrm{x}-1$ ].
6. a) The arithmetic decoding process is the reverse of the encoding procedure. Decode the message 0.23355 given the coding model

| Symbol | Probability |
| :---: | :---: |
| a | 0.2 |
| e | 0.3 |
| i | 0.1 |
| o | 0.2 |
| u | 0.1 |
| l | 0.1 |

b) A binary image contains straight lines oriented horizontally, vertically, at $45^{\circ}$, and at $-45^{\circ}$. Give a set of $3 \times 3$ masks that can be used to detect 1 -pixel breaks in these lines. Assume that the intensities of the lines and background are 1 and 0 , respectively.
7. a) Suppose that an image $f(x, y)$ is convolved with a mask of size $n \times n$ (with cofficients $1 / n^{2}$ ) to produce a smoothed image $\bar{f}(x, y)$.
i) Derive an expression for edge strength (edge magnitude) of the smoothed image as a function of mask size. Assume for simplicity that n is odd and that edges are obtained using the partial derivatives

$$
\partial \overline{\mathrm{f}} / \partial \mathrm{x}=\overline{\mathrm{f}}(\mathrm{x}+1, \mathrm{y})-\overline{\mathrm{f}}(\mathrm{x}, \mathrm{y}) \text { and } \partial \overline{\mathrm{f}} / \partial \mathrm{y}=\overline{\mathrm{f}}(\mathrm{x}+1, \mathrm{y})-\overline{\mathrm{f}}(\mathrm{x}, \mathrm{y}) .
$$

ii) Show that the ratio of the maximum edge strength of the smoothed image to the maximum edge strength of the orginal is $1 / \mathrm{n}$. In other words, edge strength is inversely proportional to the size of the smoothing mask.
b) Explain how the MPP algorithm behaves under the following conditions:
i) 1-pixel wide, 1-pixel deep indentations.
ii) 1-pixel wide, 2 -or- more pixel deep indentations.
iii) 1-pixel wide, 1-pixel longprotrusions.
iv) 1-pixel wide, n-pixel long protrusions.
8. a) Find an expression for the signature of each of the following boundaries, and plot the signatures.
i) An equilateral triangle
ii) A rectangle
iii) An ellipse
b) Consider a linear, position-invariant image degradation system with impulse response

$$
h(x-\alpha, y-\beta)=e^{-\left[(x-\alpha)^{2}+(y-\beta)^{2}\right]}
$$

Supose that the input to the system is an image cosnsiting of a line of infinitesimal width located at $\mathrm{x}=\mathrm{a}$, and modeled by $\mathrm{f}(\mathrm{x}, \mathrm{y})=\delta(\mathrm{x}-\mathrm{a})$, where $\delta$ is an impulse. Assuming no noise, what is the output image $\mathrm{g}(\mathrm{x}, \mathrm{y})$ ?
9. a) Define the terms 'Sampling' and 'Quantization' in context of digital image processing. A medical image has size $8 \times 8$ inches, the sampling reduction is 5 cycles $/ \mathrm{mm}$, calculate the number of pixels required for the medical image.
b) What do you understand by the term "Entropy" in context of any digital image? Calculate the entropy for the symbols, where probability distribution is given below:

| Symbol | Probability |
| :--- | :--- |
| 1 | 0.4 |
| 2 | 0.3 |
| 3 | 0.1 |
| 4 | 0.1 |
| 5 | 0.1 |

10. a) What is Discrete Fourier Transform (DFT)? Find DFT of the function:

$$
f(x, y)=\operatorname{Sin}\left(2 \pi u_{0} x+2 \pi v_{0} y\right)
$$

b) Apply Prewitt operators and Sobel operators for the image given below:

$$
\left[\begin{array}{lll}
\alpha_{1} & \alpha_{2} & \alpha_{3}  \tag{5}\\
\alpha_{4} & \alpha_{5} & \alpha_{6} \\
\alpha_{7} & \alpha_{8} & \alpha_{9}
\end{array}\right]
$$

