MMT-002

ASSIGNMENT BOOKLET

(Valid from 1st January, 2024 to 31st December, 2024)

M.Sc.(Mathematics with Applications in Computer Science)

LINEAR ALGEBRA

School of Sciences
Indira Gandhi National Open Universit
Maidan Garhi, New Delhi

(2024)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : ....

NAME : ....

ADDRESS : ....

....

....

COURSE CODE : ....

COURSE TITLE : ....

STUDY CENTRE : .... **DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

   We **strongly** suggest that you retain a copy of your answer sheets.

7) This assignment is valid only up to December, 2024. If you fail in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MMT-002
Assignment Code:MMT-002/TMA/2024
Maximum Marks: 100

1) Which of the following statements are true and which are false? Give reasons for your answer.

   i) If V is a finite dimensional vector space and $T: V \to V$ is a diagonalisable linear operator, then there is a basis, unique up to order of the elements, with respect to which the matrix of T is diagonal.

   ii) Up to similarity, there is a unique 3×3 matrix with minimal polynomial $(x-1)^2(x-2)$.

   iii) If λ is the eigenvalue of a matrix A with characteristic polynomial $f(x)$, $(x-\lambda)^k \mid f(x)$ and $(x-\lambda)^{k+1} \nmid f(x)$, then the geometric multiplicity of λ is at most k.

   iv) If $\rho(A) = 1$, then $A^k \to \infty$ as $k \to \infty$.

   v) If N is nilpotent, e^N is also nilpotent.

   vi) The sum of two normal matrices of the order n is normal.

   vii) If P and Q are positive definite operators, $P + Q$ is a positive definite operator.

   viii) Generalised inverse of a $n \times n$ matrix need not be unique.

   ix) All the entries of a positive definite matrix are non-negative.

   x) The SVD of any 2×3 matrix is unique. (20)

2) a) Let $T: \mathbf{C}^2 \to \mathbf{C}^2 : T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y-iz \\ 2y+iz \\ ix+z-2z \end{bmatrix}$. Find $[T]_B$, $[T]_{B'}$ and P where (10)

$$B = \left\{ \begin{bmatrix} 0 \\ i \\ 0 \end{bmatrix}, \begin{bmatrix} i \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}, B' = \left\{ \begin{bmatrix} 1 \\ -i \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \right\}, [T]_{B'} = P^{-1}\,[T]_B\,P.$$

   b) If C and D are $n \times n$ matrices such that $CD = -DC$ and D^{-1} exists, then show that C is similar to $-D$. Hence show that the eigenvalues of C must come in plus-minus pairs. (2)

   c) Can A be similar to $A + I$? Give reasons for your answer. (3)

3) Find the Jordan canonical form J for

$$B = \begin{bmatrix} -1 & 0 & -2 & -4 \\ 2 & 1 & 2 & 4 \\ -4 & 2 & -1 & -4 \\ 2 & -1 & 1 & 3 \end{bmatrix}.$$

Also, find a matrix P such that $J = P^{-1}BP$. (15)

4) a) Let M and T be a metro city and a nearby district town, respectively. Our government is trying to develop infrastructure in T so that people shift to T. Each year 15% of T's population moves to M and 10% of M's population moves to T. What is the long term effect of on the population of M and T? Are they likely to stabilise? (5)

   b) Solve the following system of differential equations: (10)

$$\frac{dy(t)}{dt} = Ay(t) \text{ with } y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \text{ where } A = \begin{bmatrix} 2 & -5 & -11 \\ 0 & -2 & -9 \\ 0 & 1 & 4 \end{bmatrix}$$

5) a) Let

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -1 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix}.$$

Find a unitary matrix U such that U^*AU is upper triangular. (5)

b) Use least squares method to find a quadratic polynomial that fits the following data: (5)

$(-2, 15.7), (-1, 6.7), (0, 2.7), (1, 3.7), (2, 9.7)$.

6) a) Check which of the following matrices is positive definite and which is positive semi-definite:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Also, find the square root of the positive definite matrix. (10)

b) Find the QR decomposition of the matrix (5)

$$\begin{bmatrix} 2 & -2 & 1 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

7) Find the SVD of the following matrices: (10)

i) $\begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ ii) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$