MMT-007

ASSIGNMENT BOOKLET

(Valid from 1st January, 2023 to 31st December, 2023)

M.Sc. (Mathematics with Applications in Computer Science)

DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

School of Sciences
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(2023)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: …………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2023. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MMT-007
Assignment Code: MMT-007/TMA/2023
Maximum Marks: 100

1. State whether the following statements are True or False. Justify your answer with the help of a short proof or a counter-example: (5×2=10)

   i) Initial value problem:

   $$\frac{dy}{dx} = \frac{y-1}{x}$$

   $y(0) = 1$ has a unique solution.

   ii) The second order Runge-Kutta method when applied to IVP $y' = -100y,\ y(0) = 1$ will produce stable results for $0 < h < \frac{1}{50}$.

   iii) If Fourier cosine transform of $f(x)$ is:

   $$F_c(n) = \frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2}$$

   where $0 \le x \le 1$, then:

   $$f(x) = 1 + 2\sum_{n=1}^{\infty} \frac{\cos\left(\frac{2n\pi}{3}\right)}{(2n+1)^2} \cos n\pi x.$$

   iv) For the differential equation $x^2(x-4)^2 y''(x) + 3xy'(x) - (x-4)y = 0$, $x = 0$, is a regular singular point and $x = 4$, is an irregular singular point.

2. a) Find the power series solution of the equation:

   $$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2+2)y = 0$$

   about its singular point. (5)

   b) Construct Green's function for the following boundary value problem:

   $$\frac{d^2y}{dx^2} + 9y = 0$$

   with $y(0) = y(1) = 0$. (5)

3. a) Find the solution of $\nabla^2 u = 0$ in R subject to R : triangle $0 \le x \le 1, 0 \le y \le 1$,

$0 \le x + y \le 1$ and $u(x, y) = x^2 - y^2$ on the boundary of the triangle. Assume $h = \frac{1}{4}$ and use five-point formula. (5)

b) Show that the method:

$$y_{i+1} = \frac{4}{3} y_i - \frac{1}{3} y_{i-1} + \frac{2h}{3} y'_{i+1}$$

is absolutely stable when applied to the equation $y' = \lambda y,\ \lambda < 0$. (3)

c) Evaluate $L^{-1}\left\{\frac{1}{(s+1)\,(s^2+1)}\right\}$, using convolution theorem. (2)

4. a) Heat conduction equation $u_t = u_{xx}$ is approximated by the method:

$$u_m^{n+1} - u_m^{n-1} = \frac{2k}{h^2}\,\delta_k^2\, u_m^n.$$

Find the order of the method and investigate the stability of this method using Von Neumann method. (5)

b) Using second order finite differences method with $h = \frac{1}{2}$, obtain the system of equations for y_0, y_1 and y_2 for solving the boundary value problem:

$$y'' - 5y' + 6y = 3$$

with $y(0) - y'(0) = -1$ and $y(1) + y'(1) = 1$. (5)

5. a) Given $\frac{dy}{dx} = x - y^2$, $y(0.2) = (0.02)$. Find $y(0.4)$ by using modified Euler's method, correct to two decimal places, taking $h = 0.2$. (5)

b) Determine an appropriate Green's function either by using the method of variation of parameters or otherwise, for the following boundary value problem:

$$-(y''(x) - y(x)) = \frac{2}{1+e^{-x}},\ y(0) = y(1) = 0. \quad (5)$$

6. a) Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ 0 \le x \le 1$

$u(x, 0) = \sin \pi x,\ 0 \le x \le 1$ $\frac{\partial u}{\partial t}(x, 0) = 0,\ u(0, t) = u(1, t) = 0,\ t > 0$ by using second order explicit method with $h = \frac{1}{4},\ r = \frac{1}{3}$. Integrate for one time step. (4)

b) Prove that

$$\frac{d}{dx}(J_n^2(x)) = \frac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)]. \quad (4)$$

c) Given

$$\frac{dy}{dx} = -300y, \ y(0) = 1.$$

Determine the value of h so that the second order Runge-Kutta method applied to be IVP produces stable results. (2)

7. a) Solve the following differential equation by power series method about $x = 0$:

$$(1 - x^2)y''(x) - 2xy'(x) + 2y(x) = 0. \quad (5)$$

b) Find the Fourier sine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2. \\ 0, & x > 2 \end{cases} \quad (5)$$

8. a) Using Laplace transform technique, solve the following initial value problem:

$$\frac{dx}{dt} + \frac{dy}{dt} = t, \ \frac{d^2x}{dt^2} - y = e^{-t},$$

$$x(0) = 0, \ y(0) = 0, \ \frac{dx}{dt} = 0, \ \text{for } t = 0. \quad (6)$$

b) Using the recurrence relation

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x),$$

prove that

$$(2n+1)^2 \int_{-1}^{1} x^2 P_n^2(x)dx = \frac{2(n+1)^2}{(2n+3)} + \frac{2n^2}{(2n-1)}. \quad (4)$$

9. a) Using the Crank Nicolson method, integrate upto one time level for the solution of the initial value problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x2}, \ 0 < x < 1$$

with $u(x, 0) = \sin 2\pi x,$

$u(0, t) = 0 = u(1, t)$

with $h = \frac{1}{3}$ and $\lambda = \frac{1}{6}.$ (5)

b) Using the substitution $z\sqrt{x}$, reduce the equation:

$$xy'' + y' + \frac{y}{4} = 0$$

to Bessel's equation. Hence find its solution. (3)

c) If H_n is a Hermite polynomial of degree n, then show that:

$$H_n^n = 4n(n-1)H_{n-2}.$$ (2)

10. a) Using Laplace transform, solve the p.d.e.:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ x > 0,\ t > 0$$

subject to the conditions:

$$u(0, t) = 10\sin 2t,$$

$$u(x, 0) = 0,$$

$$u_x(x, 0) = 0\ \lim_{x\to\infty} u(x, t) = 0.$$ (6)

b) If $f'(x_k)$ is approximated by:

$$f'(x_k) = a\,f(x_{k+1}) + bf'(x_{k+1}),$$

find the values of a and b. What is the order of approximation? (4)