MMT-004

ASSIGNMENT BOOKLET

(Valid from 1st January, 2023 to 31st December, 2023)

M.Sc. (Mathematics with Applications in Computer Science)
Computer Graphics (MMT-004)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi,
New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. …………………………..…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved..
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2023. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment (MMT – 004)

Course Code: MMT-004
Assignment Code: MMT-004/TMA/2023
Maximum Marks: 100

1. State whether the following statements are True or False. Give reasons for your answers. (10)

   a) The function $\varphi(x) = \frac{1}{x}, 3 \le x \le 4$ is not uniformly continuous.

   b) A complete metric space is a countable collection of nowhere dense sets.

   c) The function $f : \mathbf{R} \to \mathbf{R}^2$ given by $f(x, y) = (x, x|x|)$ is differentiable at 0.

   d) Any Lebesgue intergrable function is always Riemann integrable.

   e) The image of any connected set in $\mathbf{R}^2$ under the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = x^2 + y^2$ is connected.

2. a) Let A and B be non-empty disjoint closed subsets of a metric space (X, d). Show that there exist open sets $U \supset A$ and $V \supset B$ such that $U \cap V = \phi$.. (3)

   b) Define saddle points. Compute the saddle points of the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = (y - x^2)(y - 2x^2)$. (3)

   c) State the Lebesgue dominated convergence theorem. Find $\lim_{n\to\infty} \int_0^\infty \frac{\sin x}{1 + nx^2} dx$.. (4)

3. a) Define components in a metric space. What are all the components of the set of all non-zero real numbers under the (3)

   i) usual metric on $\mathbf{R}$, and
   ii) the discrete metric on $\mathbf{R}$?

   b) Find the directional derivation of the function $f : \mathbf{R}^4 \to \mathbf{R}^4$ defined by

   $$f(x, y, z, w) = (x^2 y,\ xyz,\ x^2 + y^2, zw)$$

   at $(1, 2, -1, -2)$ in the direction $\sim = (1, 0, -2, 2)$. (4)

   c) Define measurable sets in $\mathbf{R}$. Prove that intervals are measurable. (3)

4. a) If $f : X \to Y$ is a continuous map between metric spaces X and Y and K is a compact subset of X, then show that f(K) is compact. (3)

   b) Find the Taylor series expansion of the function f given by

   $$f(x, y) = x + 2y + xy - x^2 - y^2$$

about the point $(1,1)$. (4)

c) Let $f, g \in L'(\mathbf{R})$, define convolution $f * g$ of f and g. Show that if either f or g is bounded, then the convolution $f * g$ exists for all x in $\mathbf{R}$ and is bounded in $\mathbf{R}$. (3)

5. a) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences in a metric space (X, d). Show that the sequence $\{d(x_n, y_n)\}$ converges in $\mathbf{R}$. (3)

b) Consider the function $f : \mathbf{R}^3 \to \mathbf{R}$ given by

$$f(x, y, z) = x^2 + y^3 - xy \sin z.$$

Prove that the equation $f(x, y, z) = 0$ defines a unique continuously differentiable function of near $(1, -1)$ such that $g(1, -1) = 0$. (3)

c) Define and give an example for each of the following concepts in the context of signals and systems: (4)

i) A stable system
ii) A time-varying system