MMT-009

ASSIGNMENT BOOKLET

(Valid from 1st January, 2022 to 31st December, 2022)

M.Sc. (Mathematics with Applications in Computer Science)
Mathematical Modelling (MMT-009)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
2022

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved..

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2022. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.

We wish you good luck.

Assignment (MMT-009)

Course Code: MMT-009
Assignment Code: MMT-009/TMA/2022
Maximum Marks: 100

1. a) Return distributions of the two securities are given below: (7)

Return		Probabilities
X	Y	$p_{xj} = p_{yj} = p_j$
0.16	0.14	0.33
0.12	0.08	0.25
0.08	0.05	0.17
0.11	0.09	0.25

Find which security is more risky in the Markowitz sense. Also find the correlation coefficient of securities X and Y.

b) Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz? (3)

Portfolio	Expected return	Standard deviation
W	9%	21%
X	5%	7%
Y	15%	36%
Z	12%	15%

2. a) Let $P = (w_1, w_2)$ be a portfolio of two securities X and Y. Find the values of w_1 and w_2 in the following situations: (6)

i) $\rho_{xy} = -1$ and P is risk free.

ii) $\sigma_x = \sigma_y$ and variance P is minimum.

iii) Variance P is minimum and $\rho_{xy} = -0.5, \sigma_x = 2$ and $\sigma_y = 3$.

b) Tumour is developing from the organ of a human body with concentration 3.2×10^9 with growth and decay control parameters 9.2 and 2.7 respectively. In how many days the size of the tumor will be twice? (4)

3. a) Companies considering the purchase of a computer must first assess their future needs in order to determine the proper equipment. A computer scientist collected data from seven similar company sites so that computer hardware requirements for inventory management could be developed. The data collected is as follows:

Customer Orders (in thousands)	Add-delete items (in thousands)	CPU time (in hours)
123.5	2.108	141.5
146.1	9.213	168.9
133.9	1.905	154.8
128.5	0.815	146.5
151.5	1.061	172.8
136.2	8.603	160.1

92.0	1.125	108.5

i) Find a linear regression equation that best fit the data. (4)
ii) Estimate the error variance for the regression model obtained in i) above. (2)

b) Let $G(t)$ be the amount of the glucose in the bloodstream of a patient at time t. Assume that the glucose is infused into the bloodstream at a constant rate of $k\ g/\min$. At the same time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of the glucose present. If at $t = 0,\ G = G(0)$ then (4)

i) formulate the model.
ii) find $g(t)$ at any time t.
iii) discuss the long term behavior of the model.

4. a) Let the returns outcome on the two securities A and B of a company be as given below: (6)

Possible returns (%) on A: 2, 5, 4, 3, 2, 7, 8, 3, 5

Possible returns (%) on B: 7, 10, 13, 8, 7, 12, 10, 6

Find the probability of getting (i) 3% return on the security A, (ii) 8% return on the security B; and (iii) 15% return on B.

b) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by $\phi(c) = \frac{3-2c}{2(1-2c)};\ c \neq \frac{1}{2}$ together with initial conditions $c = 20\times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at $t = 20$ days. (4)

5. a) In a species of animals a constant fraction of the population $\alpha = 5.3$ are born each breeding season and a constant fraction $\beta = 4.97$ die. Formulate a difference equation for the population and find out the number of individuals after fifteen seasons given that the initial number is 987. Find the closed form solution of the formulated difference equation. If the growth rate of the population is represented by r then interpret the solution obtained when i) $r > 0$ and ii) $r < 0$. (6)

b) A model for insect populations leads to the difference equations (9)

$$N_{k+1} = \frac{\lambda N_k}{1 + a N_k}$$

where λ and a are positive constants.

i) Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.

ii) Plot the graph of $R(N_k)$ as a function of N_k.

iii) Express the intrinsic growth rate r and the carrying capacity K, for this model, in terms of the parameters, a and λ.

iv) Find the steady-state solution of this model and analyse the solution.

7. Consider a discrete model given by (10)

$$N_{t+1} = \frac{r N_t}{1 + b N_{t-1}^2} = f(N_t),\ \ r > 1.$$

Investigate the linear stability about the positive steady state N^* by setting $N_t = N^* + n_t$. Show that n_t satisfies the equation

$$n_{t+1} - n_t + 2(r-1)r^{-1}n_{t-1} = 0\,.$$

Hence show that $r = 2$ is a bifurcation value and that as $r \to 2$ the steady state bifurcates to a periodic solution of period 6.

6. a) Do the stability analysis of the following competing species system of equations with diffusion and advection (10)

$$\frac{\partial N_1}{\partial t} = a_1N_1 - b_1N_1N_2 + D_1\frac{\partial^2 N_1}{\partial x^2} - v_1\frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = -d_1N_2 + C_1N_1N_2 + D_2\frac{\partial^2 N_2}{\partial x^2} - V_2\frac{\partial N_2}{\partial x}, \quad 0 \le x \le L$$

where V_1 and V_2 are advection velocities in x direction of the two populations with densities N_1 and N_2 respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion coefficients. The Initial and boundary conditions are:

$$N_i(x,0) = f_i(x) > 0,\ 0 \le x \le L,\ i = 1, 2\cdots$$

$$N_i = \overline{N}_i \text{ at } x = 0 \text{ and } x = L\ \forall\, t, i = 1, 2\cdots$$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations.
Interpret the solution obtained and also write the limitations of the model.

b) Do the stability analysis of the following model which is formulated to study the effect of toxicant on one competing species where the environmental toxicant concentration is being taken to change w.r.t. time. (5)

$$\frac{dN_1}{dt} = r_1N_1 - \alpha_1N_1N_2 - d_1C_0N_1$$

$$\frac{dN_2}{dt} = r_2N_2 - \alpha_2N_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dP}{dt} = Q - hp - kPN_1 + gC_0N_1\,.$$

along with the initial conditions.

$$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0,\ P(0) = P_0 > 0$$

Here,

$N_1(t)$ = Density of prey population

$N_2(t)$ = Density of predator population

$C_0(t)$ = Concentration of the toxicant in the individuals of the prey population

P = Constant environmental toxicant concentration.

α_1, α_2 are the predation rates, r_1, r_2 are the growth rates or birth rates, d_1 is the death rate due to C_0, m_1 is the depuration rate, Q, h, k, g are positive rate constants.

8. a) The population consisting of all married couples is collected. The data showing the age of 12 married couples is as follows:

Husband's age (years)	Wife's age (years)	Husband's age (years)	Wife's age (years)
30	27	51	50
29	20	48	46
36	34	37	36
72	67	50	42
37	35	51	46
36	37	36	35

i) Draw a scatter plot of the data (2)

ii) Write two important characteristics of the data that emerge from the scatter plot. (2)

iii) Fit a linear regression model to the data and interpret the result in terms of the comparative change in the age of husband and wife. (3)

iv) Calculate the standard error of regression and the coefficient of determination for the data. (3)

9. a) Consider the data showing observations on the quantity demanded of a certain commodity depending on commodity price and consumers' income: (5)

Quantity demanded	Price (in ₹)	Income (in ₹)
100	5	1000
75	7	600
80	6	1200
70	6	500
50	8	300
65	7	400
90	50	1300
100	4	1100
110	3	1300
60	9	300

Find a multiple regression equation that best fits the data.

b) Consider the budworm population dynamics governed by the equation (5)

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - x,$$

where k, the carrying capacity, and r, the birth rate of the budworm population, are positive parameters. Find out the steady states and use the perturbation to do the stability analysis of the equation for $0 < r < 1$.