MMT-003

ASSIGNMENT BOOKLET

M.Sc. (Mathematics with Applications in Computer Science)

ALGEBRA

(Valid from 1st July, 2022 to 30th June, 2023)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2022-23)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: …………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto June, 2023. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by June, 2023, then you need to get the assignment for 2023-24, and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

(To be done **after** studying Blocks 1-4.)

Course Code: MMT-003
Assignment Code: MMT-003/TMA/2022-23
Maximum Marks: 100

1. Which of the following statements are true? Give reasons for your answers. Marks will only be given for valid justification of your answers.

   i) If G is a finite abelian group and p is a prime factor of $o(G)$, then the number of Sylow p-subgroups of G is a prime.

   ii) The minimum polynomial of $5^{1/3}$ over $\mathbb{Q}$ is $x^{1/3}$.

   iii) $\mathbb{Z}_{mn} \simeq \mathbb{Z}_m \times \mathbb{Z}_n \ \forall \ m, n \in \mathbb{N}$.

   iv) If G is a finite group and $m | o(G)$, $m \in \mathbb{N}$, then G has a subgroup of order m.

   v) If β_1 and β_2 are two 10^{th} roots of unity, then $\mathbb{Q}(\beta_1) = \mathbb{Q}(\beta_2)$.

   vi) There exists an extension field of $\mathbb{Z}_3$ of order 25.

   vii) Every group of order 18 has a normal subgroup of order 2.

   viii) If I and J are ideals of a ring R, then $IJ = I \cap J$.

   ix) If $f : R \to S$ is a ring homomorphism and I is an ideal of R, then $f(I)$ is an ideal of S.

   x) Every prime ideal of an integral domain is a maximal ideal. (20)

2) a) Let G be a group and let $H \le G, K \le G, o(H) = o(K) = p$, a prime. Show that either $H \cap K = \{e\}$ or $H = K$. Is this result still true if p is not a prime? Justify your answer. (5)

   b) Let G be the group of all rigid motions of a plane and S be the set of all rectangles in the plane. Show that G acts on S. Also obtain the orbit and stabiliser of a **square** under this action. (7)

   c) Let G be a finite group and H be a normal subgroup of G. Prove that $|H| = \sum\{|C_x| \,|\, x \in H, \text{the } C_x \text{ are all distinct}\}$. (3)

3) a) Find the number of Sylow 5-subgroups, Sylow 7-subgroups and Sylow 2-subgroups A_5 has. (10)

   b) Let G_1 and G_2 be finite groups such that p divides $|G_1|$ and $|G_2|$. Prove that the Sylow p-subgroups of $G_1 \times G_2$ are precisely of the form $P_1 \times P_2$, where P_1 and P_2 are Sylow p-subgroups of G_1 and G_2, respectively. (5)

4. a) Write $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 3 \\ 5 & 5 & 1 \end{bmatrix}$ as a product of $O(3)$ and an element of $B_3(\mathbb{R})$. (5)

b) Show that $SU(2)$ and S^3 are structurally the same. (5)

5. a) Find all the possible abelian groups, up to isomorphism, of order 900. (5)

b) Construct the free group on the set $\{\alpha, \beta, \gamma\}$. Further, check if it is a free abelian group or not. (5)

6. a) Check whether or not the ring $R = \mathbb{Z}_3[x] / < x^6 - 1 >$

i) is finite;

ii) has zero divisors;

iii) has nilpotent elements. (5)

b) Give two distinct rings whose quotient field is $\{a + ib \mid a, b \in \mathbb{Q}\}$. Justify your answer. (5)

c) Prove that $\mathbb{R}^{n+3} / \mathbb{R}^n$ and $\mathbb{R}^3$ are isomorphic rings $\forall\ n \in \mathbb{N}$. (5)

7. a) Check whether or not $\mathbb{Z}[\sqrt{-7}]$ is a Euclidean domain. (5)

b) Use the division algorithm to find the inverse of $\overline{18}$ in $\mathbb{Z}_{35}$. (5)

8. i) Let G be a group of automorphisms of a field K. Is the fixed field K^G a subfield of K ? Why, or why not?

ii) Find K^G, where $K = \mathbb{Q}(i, \sqrt{3}), G = G(K/\mathbb{Q})$. (5)