MMTE-001

ASSIGNMENT BOOKLET

M.Sc.(Mathematics with applications to computer science)

Graph Theory

(Valid from 1st January, 2021 to 31st December, 2021)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(For January 2021 cycle)

Dear Student,

Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. A weightage of 30%, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

	ROLL NO. :..........................
	NAME :..........................
	ADDRESS :..........................
	....
	....
COURSE CODE :	....
COURSE TITLE :	
STUDY CENTRE :	**DATE**..........................

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

7) This assignment is valid only up to 31st December, 2021. If you fail in this assignment or fail to submit it by 31st December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

9) **We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

Assignment

Course Code: MMTE-001
Assignment Code: MMTE-001/TMA/2021
Maximum Marks: 100

1) State whether the following statements are **true** or **false**. Justify your answers with a short proof or a counter-example (20)

   i) Every even graph is regular.

   ii) The independence number of a cycle is equal to its matching number.

   iii) The diameter of $K_{m,n}$ is 2 for all m and n.

   iv) If v is a leaf of a graph and u is the vertex adjacent to v, then $\varepsilon(v) = \varepsilon(u) + 1$.

   v) Every path is a trail.

   vi) The diameter of every tree is equal to the twice of the radius of the tree.

   vii) The number of pairwise nonisomorphic trees on 4 vertices is 8.

   viii) If G is a 2-edge connected graph, then G is 2-connected.

   ix) If a graph G has no vertex of degree 5 or more, then $\chi(G) \leq 4$.

   x) Every subgraph of a planar graph is planar.

2) a) Give an example of each of the following: (4)

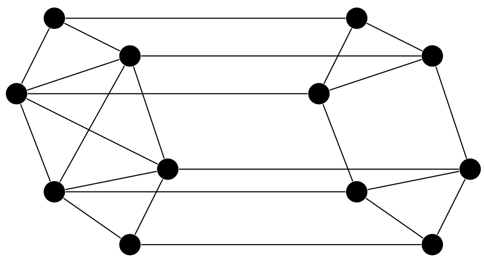
      i) a nonsimple plane graph with its dual a simple graph

      ii) a self-dual plane graph

      iii) a plane graph whose each face is bounded by a triangle

      iv) a plane graph on 8 vertices with its dual having just one vertex

   b) Is the following graph isomorphic to Q_4? If yes, give an isomorphism from Q_4 to this graph. Otherwise, find a copy of Q_3 in it. (2)



   c) If v is a cut-vertex of a graph G, then v is also a cut-vertex of every induced subgraph of G containing v. Prove or disprove. (2)

   d) Find the maximum size of an independent set, and the maximum size of a clique in the graph given above. (2)

3) a) Show that the number of leaves in an n-vertex tree with maximum degree $\Delta \geq 2$ lies between Δ and $\frac{n(\Delta-2)+2}{\Delta-1}$. (6)

   b) Give an example of each of the following: (4)

      i) A graph G with $\chi(G) > \omega(G) = 3$

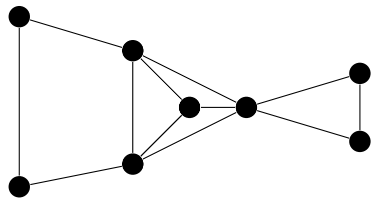
      ii) A graph G which is 4-critical.

c) Let T be a tree with at least 3 vertices. Let T' be the graph obtained from T by deleting all the leaves. Show that

i) T' is a tree,

ii) T and T' have the same centre. (3)

d) Find the diameter and radius of the following graph. (2)



4) a) Prove that K_4 is not outerplanar. (3)

b) Check whether the sequence $(5,5,4,4,3,3,3,3,1,1,1)$ is graphic or not. If it is graphic, draw a graph with this degree sequence. (3)

c) Define Mycielski's construction. Use this to obtain a graph with chromatic number 4 from K_2. (4)

5) a) Let M be a matching in a graph G such that G has no M-augmenting path. Show that M is a maximum matching. (4)

b) Let G be a bipartite graph with bipartition (X,Y). For $S \subseteq X$, let E_1 be the set of edges in G incident on some vertex in S, and let E_2 be the set of edges in G incident with some vertex in $N(S)$. Is it true in general that $E_1 \subseteq E_2$? Why? (4)

c) Prove that if G is a graph with no isolated vertices, then $\alpha'(G)+\beta'(G)=n(G)$ (4)

d) Which of the following is true ? Justify. (3)

i) Every tree has a perfect matching.

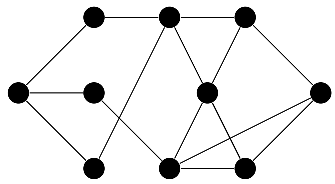
ii) Every tree has at most one perfact matching.

6) a) Let G be a connected graph with $n(G) \geq 2$. Show that G at least 2 vertices which are not cut-vertices of G. (3)

b) For the following graph G find the following: (3)

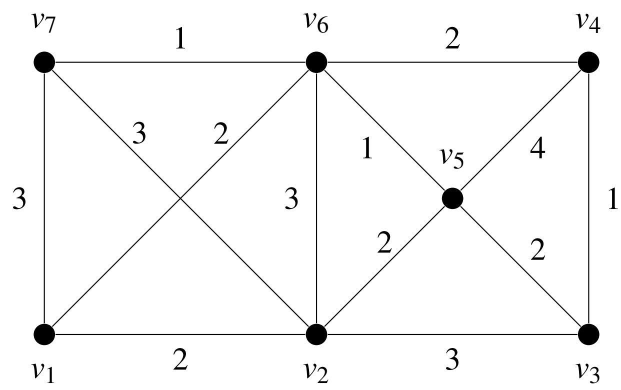
i) $\kappa(G)\kappa'(G)$

ii) a separating set S such that $|S| = \kappa(G)$



c) Prove that if a plane graph G has n vertices, e edges, f faces and k components, then $n-e+f=k+1$. (3)

d) Find a minimal spanning tree in the following weighted graph using the Prim's algorithm. (6)



7) a) Prove that a bipartite graph has a unique bipartition iff it is connected. (5)

b) Prove that if G is a simple graph with $n \geq 3$ vertices and $\binom{n-1}{2} + 2$ edges, then G is Hamiltonian. (5)

c) If G is a Hamiltonian graph, then G has no cut-vertex. True or false? Justify. (2)

d) Give an example of a graph on 8 vertices which is

i) Hamiltonian but not Eulerian.

ii) Eulerian but not Hamiltonain. (3)