MPH-004

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme

(MSCPH)

QUANTUM MECHANICS-I

Valid from 1st July, 2023 to 30th June, 2024

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2023-24)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: …..…………………………………………

NAME: ………………………………………………

ADDRESS: ……………………………………………

COURSE CODE:…………………………..

COURSE TITLE: …………………………

ASSIGNMENT CODE: …………………..

STUDY CENTRE: ……………………….. **DATE:** …………………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) **Submit the complete assignment answer sheets containing Part A and Part B, within the due date.**
6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**
7) This assignment is **valid from 1st July, 2023 to 30th June, 2024**. If you have failed in this assignment or fail to submit it by June 30, 2024, then you need to get the assignment for the year 2024-25, and submit it as per the instructions given in the Programme Guide.
8) **You cannot fill the examination form for this course** until you have submitted this assignment. For any queries, please contact: mbnewmai@ignou.ac.in, slamba@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment
QUANTUM MECHANICS-I

Course Code: MPH-004
Assignment Code: MPH-004/TMA/2023-24
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Calculate the average de Broglie wavelength of a nitrogen molecule at room temperature, given that the mass of nitrogen molecule is 4.65×10^{-26} kg. (5)

   b) A particle of mass m is constrained to move in a one-dimensional region between two infinitely high potential barriers separated by a distance L_0. Using the uncertainty principle, determine the zero-point energy of the particle. (5)

   c) The wavefunction for a particle is given by:

   $$\psi(x) = \begin{cases} N\left(L^2 - x^2\right) & -L \le x \le L \\ 0 & \text{elsewhere} \end{cases}$$

   Calculate the normalization constant N. (5)

   d) Show that $\left[\hat{L}_x \hat{L}_y, \hat{L}_z\right] = i\hbar\left(\hat{L}_x{}^2 - \hat{L}_y{}^2\right)$ (5)

2. a) Calculate the expectation values $\langle \hat{x} \rangle$ and $\langle \hat{x}^2 \rangle$ for the following odd parity state of a symmetric infinite potential well

   $$\psi(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi x}{2a}\right) \; ; \; n = 2,4,6....$$ (10)

   b) The initial wave function for a simple harmonic oscillator is

   $$\psi(x,0) = \frac{1}{2}\psi_0(x) - i\frac{\sqrt{3}}{2}\psi_2(x)$$

   where ψ_0 and ψ_2 are the normalized eigenfunctions of the simple harmonic oscillator. Determine $\psi(x,t)$ and the expectation value of $\hat{H}$ in the state $\psi(x,0)$. (10)

   c) Determine $\langle r \rangle$ for an electron in the ψ_{210} state of the hydrogen atom and show that the most probable value of r for this state is $4a_0$. (10)

PART B

3. a) Consider the two following state vectors in a vector space spanned by the orthonormal eigenkets $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$:

$$|\psi_1\rangle = 2i|\phi_1\rangle + |\phi_2\rangle - i|\phi_3\rangle$$

$$|\psi_2\rangle = i|\phi_1\rangle - 2|\phi_2\rangle$$

Determine

i) $\langle\psi_1|\psi_1\rangle$; $\langle\psi_1|\psi_2\rangle$; and $\langle\psi_2|\psi_2\rangle$

ii) The matrix elements of the operator $|\psi_1\rangle\langle\psi_2|$ (6+4)

b) Determine $\frac{d\hat{x}}{dt}$ and $\frac{d\hat{p}}{dt}$ for a particle of mass m in a gravitational field, with the Hamiltonian: $H = \frac{p^2}{2m} + mgx$. Solve the equations of motion to get $\hat{x}(t)$ and $\hat{p}(t)$ in terms of $\hat{x}(t=0) = \hat{x}(0)$ and $\hat{p}(t=0) = \hat{p}(0)$ and calculate $\left[\hat{x}(t_1), \hat{p}(t_1)\right]$. (10)

4. a) i) Show that $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$.

ii) Using the equation $\psi_{n+1}(x) = \frac{1}{\sqrt{2(n+1)}}\left[\xi - \frac{d}{d\xi}\right]\psi_n(x)$ determine $\psi_3(x)$. (5+5)

b) i) Write down the angular momentum states $|j, m_j\rangle$ for $j = \frac{5}{2}$ and the eigenvalue of $\hat{J}^2$ and $\hat{J}_z$ for each of these states.

ii) Obtain the angular momentum matrix J_y for $j = 1$. (5+5)

c) Show that for the electron

$$\hat{S}_x = \frac{\hbar}{2}\left[|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\right]$$

and

$$\hat{S}_y = \frac{i\hbar}{2}\left[-|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\right]$$ (5+5)

****