MMT-007

ASSIGNMENT BOOKLET

(Valid from 1st January, 2022 to 31st December, 2022)

M.Sc.(Mathematics with Applications in Computer Science)

DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

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THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2022)

Dear Student,
Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 20 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : ....
NAME : ....
ADDRESS : ....
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....
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COURSE CODE : ....
COURSE TITLE : ....
STUDY CENTRE : .... **DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **Solve the assignment on your own. Don't copy from your fellow students or from the internet. If you are found guilty of copying, your assignment will be disqualified and you will have to submit the assignment for the next session.**

7) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

8) This assignment is valid only up to December, 2022. If you fail in this assignment or fail to submit it by December, 2022, then you need to get the fresh assignment for the year 2023 and submit it as per the instructions given in the Programme Guide.

9) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

10) We **strongly** suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MMT-007
Assignment Code: MMT-007/TMA/2022
Maximum Marks: 100

1) State whether the following statements are true or false. Justify your answer with the help of a proof or a counter example. (10)

   i) The Lipschitz constant for the function $f(x,y) = x^3|y|$, defined on $|x| \leq 1, |y| \leq 1$ is equal to 2.

   ii) The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{3^n}$, is $\frac{1}{3}$.

   iii) $L^{-1}\left[\ln\left(\frac{S+b}{S+a}\right)\right] = \frac{1}{t}\left(e^{-at} - e^{-bt}\right)$

   iv) If the value of h that can be used so that the method produces stable results is $h < 0.04$, Euler's method is to be used to solve the initial value problem

   $$y^1 = -50y, y(0) = 2.$$

   v) The p.d.e. $\frac{\partial^2 u}{\partial x^2} + 2x\,\frac{\partial^2 u}{\partial x\,\partial y} + \left(1+y^2\right)\frac{\partial^2 u}{\partial y^2}$ is elliptic in the region $x^2 - y^2 > 1$.

2) a) Solve by the method of Laplace transform: (4)

   $$y'' + 2y' - 3y = 3, y(0) = 4, y'(0) = 7$$

   b) Find the power series solution near $x = 0$ of the differential equation (6)

   $$9x(1-x)y'' - 12y' + 4y = 0$$

3) a) Solve the following boundary-value problem by determining the appropriate Green's function via the method of variation of parameters. Express the solution as a definite integral. (5)

   $$-\left(\frac{d^2y}{dx^2} + 4y\right) = f(x), y(1) = 0, y'(0) = 0$$

   b) Given $\frac{dy}{dx} = \frac{1}{2}\left(1+x^2\right)y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21$, evaluate $y(.4)$ using Milne's predictor corrector method. (5)

4) a) Solve the initial value problem $y' = -2xy^2, y(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using the predictor-corrector method (5)

   $$P: y_{k+1} = y_k + \frac{h}{2}\left(3y_k^1 - y_{k-1}^1\right)$$
   $$C: y_{k+1} = y_k + \frac{h}{2}\left(y_{k+1}^1 + y_k^1\right).$$

   Perform two corrector iterations per step. Use the exact solution $y(x) = \frac{1}{1+x^2}$ to obtain the starting value.

b) Apply fourth order Runge-Kutta method ro solve the initial-value problem. (5)

$$\frac{dy}{dx} = x - 2y, y(0) = 1$$

to obtain y for $x = 0.1$ and $x = 0.2$.

5) a) Solve boundary value problem (5)

$$y'' = 6y^2, y(0) = 4, y(\frac{1}{2}) = 1$$

using second order finite difference method with $h = \frac{1}{4}$.

b) Using central difference approximation method solve the following boundary value problem (5)

$$y'' + y + 1 = 0, y(0) = 0, y(1) = 0,$$

taking $h = \frac{1}{2}$. Also estimate the order of the method.

6) a) Using second order finite difference method with $h = \frac{1}{3}$, solve the boundary value problem (5)

$$y'' - 3y' + 2y = 0$$
$$y(0) = 1, y(1) = 0$$

b) Given $\frac{dy}{dx} = 1 + y^2$ with $y = 0$ when $x = 0$, find $y(0.2)$ and $y(0.4)$ using fourth order Runge-Kutta formula with $h = 0.2$. (5)

7) a) Solve $\frac{\partial u}{\partial t} = 2\,\frac{\partial^2 u}{\partial x^2}$ given that $u(0, y) = 0$, (5)

$$u(5, t) = 0, u(x, 0) = \sin\,(\pi x),$$

using Laplace transforms.

b) Using Laplace transforms, solve (5)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, x > 0, t > 0$$

given that $u(0, t) = 10\ \sin 2t, u(x, 0) = 0$

$u_t(x, 0) = 0$ and $\lim_{x\to\infty} u(x, t) = 0$.

8) a) Using Fourier transforms, determine the solution of the equation (5)

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^1 z}{\partial x^1} = 0, (-\infty < x < \infty, y \geq 0),$$

satisfying the conditions

i) z and its derivatives tend to zero as $x \to \pm\infty$

ii) $z = f(x), \frac{\partial z}{\partial y} = 0$ on $y = 0$.

b) Find the solution of the boundary value problem: (5)

$$\nabla^2 u = x^2 + y^2$$
$$0 \leq x \leq 1$$
$$0 \leq y \leq 1$$

subject to the boundary conditions:

$$u = \frac{1}{12}\left(x^4 + y^4\right)$$

on the lines $x = 1, y = 0, y = 1$ and

$$12u + \frac{\partial u}{\partial x} = x^4 + y^4 + \frac{x^3}{3}$$

on $x = 0$ using the five point formula. Assume $h = \frac{1}{2}$ along both axes. Use central difference approximation in the boundary condition.

9) a) Find: (5)

$$L\left(\sin\sqrt{t}\right)$$

where L denoted Laplace transform.

Deduce the value of $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$.

b) Find the solution of the initial boundary value problem: (5)

$$u_t = u_{xx}$$
$$0 \leq x \leq 1$$
$$t > 0$$

with conditions:

$$u(x,0) = \begin{cases} 2x, & 0 \leq x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} \leq x \leq 1 \end{cases}$$

$u(0,t) = 0 = u(1,t), t > 0$ using Crank-Nicholson method with $\lambda = \frac{1}{2}$. Assume $h = \frac{1}{4}$ and interpret for one time level.

10) a) Construct Green's function for the b.v.p.: (5)

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2,\ 0 < x < \frac{\pi}{2},$$

$y(0) = 0, y\left(\frac{\pi}{2}\right) = 0.$

b) Using Inverse Fourier transform, find $f(x)$ if: (3)

$$F_c(\alpha) = \begin{cases} \left(a - \frac{\alpha}{2}\right), & \alpha \leq 2a \\ 0, & \alpha > 2a \end{cases}$$

c) Find the Laplace inverse of $\cot^{-1} s$. (2)