MMT-004

ASSIGNMENT BOOKLET

(Valid from 1st January, 2018 to 31st December, 2018)

M.Sc. (Mathematics with Applications in Computer Science)

REAL ANALYSIS (MMT-004)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2018)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. …………………………..…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved..
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2018. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2018, then you need to get the assignment for the year 2019 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment (MMT – 004)

Course Code: MMT-004
Assignment Code: MMT-004/TMA/2018
Maximum Marks: 100

1. State whether the following statements are True or False. Give reasons for your answers. (5x2=10)

   a) $\{0\} \cup \{\frac{1}{n} \mid n = 1,2,3,...\}$ is a compact set in $\mathbf{R}$.

   b) $\mathbf{R}^2 \setminus \{(1, a) : a \in \mathbf{R}\}$ has two components.

   c) If the partial derivatives of a function $f : \mathbf{R}^2 \to \mathbf{R}$ exist at (x_0, y_0), then f is differentiable at (x_0, y_0).

   d) The measure of the set of irrationals is zero.

   e) For $f, g \in L^1([a,b]), \|f - g\|$ defined as $\|f - g\| = \int_a^b (f(x) - g(x))\, dx$ is a metric on $L^1([a,b])$.

2. a) Find the stationary points of the function $f : \mathbf{R}^3 \to \mathbf{R}$ given by $f(x, y, z) = (x + y + z)^3 - 3(x + y + z) - 24xyz$ and check whether they are local extreme points. (5)

   b) Is an arbitrary union of closed sets closed? Justify your answer. (2)

   c) Prove that for a measurable function f, $\left|\int f\, dm\right| \leq \int |f|\, dm$. (3)

3. a) Let E be the open set in $\mathbf{R}^2$ given by $E = \{x \in \mathbf{R}^2 : \|x\| < 1\}$. Prove that the function $f : E \to \mathbf{R}^3$ given by $f(x_1, x_2) = (e^{x_1}, e^{x_2}, x_1 - x_2)$ belongs to $C^1(E)$. (4)

   b) Check the measurability and integrability of the following functions defined on $\mathbf{R}$. Justify your answers. (6)

      i) $f(x) = 2, \quad x = 1,2,3,4$
         $= -1, \ x = -1,-2,-3$
         $= 0, \quad$ elsewhere.

      ii) $f(x) = x + e^x$

      iii) $f(x) = \frac{5}{2}, \ x \in [0,6]$
         $= 0$ elsewhere.

4. a) Prove that the function $f:\mathbf{R}^4 \to \mathbf{R}^4$ defined by $f(x,y,z,w)=(x+2y, x^2-y^2, wz, y+w)$ is locally invertible at $(1,1,1,1)$. (3)

b) Find the interior, closure and boundary of the set $\mathbf{A}=\{90,y)\in\mathbb{R}^2: 0\le y\le 1\}$ as a subset of $\mathbf{R}^2$ with standard metric. (3)

c) Obtain the second Taylor series expansion for the function $f:\mathbf{R}^2\to\mathbf{R}$ given by $f(x,y)=x+2y+xy-x^2-y^2$ at $\left(\frac{1}{4},\frac{3}{4}\right)$. (4)

5. a) Suppose that X is a complete metric space and A is a subset of X. Show that A is totally bounded if and only if $\overline{A}$ is compact. (4)

b) Check the differentially of the following functions F at the indicated points. Also find F' wherever exists.

i) $F:\mathbf{R}^2\to\mathbf{R}$ defined by $F(x,y)=|x|+|y|$ at (1,0).

ii) $F:\mathbf{R}\to\mathbf{R}^2$ defined by $F(x)=(f_1(x), f_2(x))$ where $f_1(x)=x$ and
$$f_2(x)=\begin{cases} x\sin 1/x, & (x\ne 0)\\ 0, & (x=0)\end{cases}.$$

iii) $F:\mathbf{R}^4\to\mathbf{R}^3$ defined by $F(x,y,z,w)=(x^2+y^2+zw^2, x^2y, xyz)$ at $(1,2,-1,2)$. (6)

6. a) For $f,g\in L^1(\mathbb{R})$, define convolution $f*g$. Prove that "convolution" is commutative. (4)

b) Let f and g be the functions given by

$$f(t)=\begin{cases}\sqrt{t}, & \text{if } 0<t<1\\ 0, & \text{if } t\le 0 \text{ or } t\ge 1\end{cases}$$

$$g(t)=\begin{cases}\sqrt{1-t}, & \text{if } 0<t<1\\ 0\ , & \text{if } t\le 0 \text{ or } t\ge 1\end{cases}$$

Find $f*g$ and $g*f$ and verify that they are the same. (6)

7. a) Suppose X and Y are metric spaces with X compact. Prove that a continuous function $f:X\to Y$ is uniformly continuous. (4)

b) Find the directional derivative of the function $f:\mathbf{R}^3\to\mathbf{R}$ given by $f(x,y,z)=y^2+2xz$ in the direction $v=(0,1,z)$ at the point $(1,2,-3)$. (3)

c) Show that any continuous image of and a path connected set is path connected. (3)

8. a) Give examples of the following. Justify your choice of examples.

   i) A connected set in $\mathbf{R}$ with standard metric which is countable.

   ii) A compact set in $\mathbf{R}$ with discrete metric.

   iii) A subspace of $\mathbf{R}$ with the standard metric which is not complete with respect to the metric on it. (6)

   b) Show that an arbitrary intersection of sets which are both compact and closed is compact? Check whether the hypothesis "closed" is necessary or not. (4)

9. a) Let $C[a,b]$ denote the set of measurable functions which are continuous on [a, b]. Show that $C[a,b] \subseteq L^1[a,b]$. (3)

   b) Consider the function $f:\mathbf{R}^4 \to \mathbf{R}^2$ given by $f(x,y,z,w) = (x^2 - y^2, z^2 - w^2)$. Check whether the second order partial derivatives of this function exists at $(1,0,-1,2)$. (3)

   c) Find the Fourier series of the function f defined by

$$f(x) = \begin{cases} -x^2, & -\pi < x \le 0 \\ x^2, & 0 < x < \pi \end{cases}. \qquad (4)$$

10. a) Find the maximum value of the function $f:\mathbf{R}^n \to \mathbf{R}$ given by $f(x_1,....,x_n) = (x_1 \times x_2 ... \times x_n)^2$ subject to the constraint $\sum x_i^2 = 1$. Interpret it geometrically for $n = 2,3$. (6)

   b) Verify implicit function theorem for the following function $f:\mathbf{R}^4 \to \mathbf{R}^2$ given by $f(x,y,z,w) = (x + 2y + z^2 - w^2, -x - y + z^2 + w^2)$ at $(0,0,0,0)$. (4)