MMT-002

ASSIGNMENT BOOKLET

Masters in Mathematics with Applications to Computer Science

Linear Algebra

(Valid from 1st January, 2018 to 31st December, 2018.)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2018)

Dear Student,
Please read the section on assignments in the Programme Guide that we sent you after your enrolment. As you may know already from the programme guide, the continuous evaluation component has 30% weightage. This assignment is for the continuous evaluation component of the course.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO : ....
NAME : ....
ADDRESS : ....
....
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COURSE CODE : ....
COURSE TITLE : ....
STUDY CENTRE : .... **DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) Write all the answers in your own words. Do not copy from the internet, from your fellow students or from any other source. Your counsellor may ask you to explain any answer in your own words.

7) This assignment is valid only up to December, 2018. If you fail in this assignment or fail to submit it by December, 2018, then you need to get the assignment for 2019 and submit it as per the instructions given in the Programme Guide.

We **strongly** suggest that you retain a copy of your answer sheets.

Wish you good luck.

Assignment

Course Code: MMT-002
Assignment Code: MMT-002/TMA/2018
Maximum Marks: 100

1) Which of the following statements are true and which are false? Give reasons for your answer.

i) The matrices $\begin{bmatrix}1&0&0\\0&1&1\\0&0&1\end{bmatrix}$ and $\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}$ are similar.

ii) If A and B are $n\times n$ matrices such that $\mathrm{tr}(A)=\mathrm{tr}(B)$, then A and B must be similar.

iii) If T is a diagonalisable linear operator, then the geometric multiplicity of each of its eigenvalues is 1.

iv) The matrix $\begin{bmatrix}3&1&0\\0&3&0\\0&0&2\end{bmatrix}$ is diagonalisable.

v) The only diagonalisable, 3×3 matrix with characteristic polynomial $(x-3)^2(x-1)$ is $\begin{bmatrix}3&0&0\\0&3&0\\0&0&1\end{bmatrix}$.

vi) There is a unique $n\times n$ matrix with spectral radius n.

vii) If D and N are diagonalisable and nilpotent matrices, respectively, then $DN=ND$.

viii) If a 2×2 square matrix is unitarily diagonalisable, it is self-adjoint or unitary.

ix) If A is a symmetric matrix and A^3 is positive definite, A is also positive definite.

x) The SVD for a column matrix is a 1×1. (20)

2) a) Let $T\colon \mathbf{C}^2\to\mathbf{C}^2\colon T\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x+2y-iz\\2y+iz\\x+z-2z\end{bmatrix}$. Find $[T]_B$, $[T]_{B'}$ and P where (10)

$$B=\left\{\begin{bmatrix}0\\i\\0\end{bmatrix},\begin{bmatrix}i\\i\\-i\end{bmatrix},\begin{bmatrix}0\\0\\2\end{bmatrix}\right\}, B'=\left\{\begin{bmatrix}1\\-i\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix},\begin{bmatrix}1\\i\\0\end{bmatrix}\right\}, [T]_{B'}=P^{-1}[T]_BP.$$

b) If C and D are $n\times n$ matrices such that $CD=-DC$ and D^{-1} exists, then show that C is similar to $-D$. Hence show that the eigenvalues of C must come in plus-minus pairs. (2)

c) Can A be similar to $A+I$? Give reasons for your answer. (3)

3) a) Let $t\colon \mathbf{R}^3\to\mathbf{R}^3: T\begin{bmatrix}x\\y\\c\end{bmatrix}=\begin{bmatrix}x+2y\\3x+z\\z\end{bmatrix}$. Obtain the generalised eigenspaces of T of order 2 and 3, for each of the eigenvalues of T.

b) Find the Jordan form and the matrix P for $B=\begin{bmatrix}1&-1&0&-1\\0&2&0&1\\-2&1&-1&1\\2&-1&2&0\end{bmatrix}$, where $J=P^{-1}JP$. (15)

c) Let $A=\begin{bmatrix}-2&3&0\\1&-1&1\\0&-1&2\end{bmatrix}$. Find an invertible matrix such that $P^{-1}AP$ is in block diagonal form. (10)

4) a) Let M and T be a metro city and a nearby district town, respectively. Our government is trying to develop infrastructure in T so that people shift to T . Each year 10% of T's population moves to M and 1% of M's population moves to T . What is the long term effect of this on the populations of M and T ? Are they likely to stabilise? (5)

b) Solve the following system of differential equations:

$$\frac{dy(t)}{dt} = Ay(t) \text{ with } y(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \text{ where } A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (6)

c) Find the QR decomposition of the matrix (2)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

d) Find e^A where (2)

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

5) Find a unitary matrix U such that U^*AU is upper triangular, where

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 1 & 0 \\ -6 & 2 & -1 \end{bmatrix}$$

Hence obtain an orthonormal basis for the linear operator T on $\mathbf{R}^3$ with respect to which the matrix of T is upper triangular. Here T is defined by A w.r.t. the standard orthonormal basis on $\mathbf{R}^3$. (10)

6) Find the SVD decomposition of the following matrices: (10)

a) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 4 \end{bmatrix}$.

7) Find the spectral decomposition of the following matrix: (5)

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$