BMTC-133

ASSIGNMENT BOOKLET

REAL ANALYSIS

Valid from 1st January, 2025 to 31st December, 2025

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

Course Code: BMTC-133
Assignment Code: BMTC-133/TMA/2025
Maximum Marks: 100

1. Which of the following statements are true or false? Give reasons for your answers in the form of a short proof or counter-example, whichever is appropriate: (10)

   i) Every infinite set is an open set.

   ii) The negation of $p \wedge \sim q$ is $p \to q$.

   iii) -1 is a limit point of the interval $]-2,1]$.

   iv) The necessary condition for a function f to be integrable is that it is continuous.

   v) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x-2| + |3-x|$ is differentiable at $x = 5$.

2. a) Test the following series for convergence. (6)

   (i) $\sum_{n=1}^{\infty} n\, x^{n-1}, x > 0$.

   (ii) $\sum_{n=1}^{\infty} \left[\sqrt{n^4+9} - \sqrt{n^4-9}\right]$

   b) Prove that the sequence $(a_n)_{n\in\mathbb{N}}$, where $a_n = \dfrac{3^2}{x^2+2^2}$, is Cauchy. (4)

3. a) Show that the set: (5)

$$S = \left\{ \frac{1}{n} + \frac{1+(-1)^n}{3} : n \in \mathbb{N} \right\}$$

   is not closed.

   b) Test the following series for convergence: (5)

$$\frac{1}{3.4} + \frac{\sqrt{2}}{5.6} + \frac{\sqrt{3}}{7.8} + \ldots\ldots \text{to } \infty$$

4. a) Prove that a function $f : S \to S$ (where S is a finite non-empty set) is injective if it is surjective. (3)

   b) Disprove the statement: (2)

$$"(x+y)^n = x^n + y^n \ \forall n \in \mathbb{N}, x, y \in \mathbb{Z}"$$

   by providing a suitable counter-example.

c) Find the supremum and infimum of the set: (5)

$$S=\left\{\frac{1}{n-1}: n\geq 2\right\}.$$

5. a) Show that $[a,\infty)$ is closed set. (3)

b) Write the following statement, and its negation, using logical quantifiers. Also interpret its negation in words. (3)

$$\exists\, x\in\mathbb{R} \text{ such that } x-\frac{1}{3}>0.$$

c) Let x and y be two real numbers such that $x<y$. Show that there exists an irrational number λ such that (4)

$$x<\lambda<y.$$

6. a) Prove that between any two real roots of $e^{x}\cos 2x=2,$ there is at least one real root of

$e^{x}\sin 2x=1.$ (5)

b) Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by: (5)

$$f(x)=\begin{cases} x^{3}\cos\left(\dfrac{1}{2x}\right), & \text{if } x\neq 0\\ 0 & , \text{if } x=0\end{cases}$$

Check whether f' is continuous on $\mathbb{R}$.

7. a) Apply the Cauchy's integral test to evaluate: (4)

$$\lim_{x\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+\ldots+\frac{1}{n}\right]$$

b) For $x\in[0,2]$ and $n\in\mathbb{N}$, define $f_n(x)=3x^2+\dfrac{2x}{n}$. Find the limit function 'f' of the sequence $(f_n)_{n\in\mathbb{N}}$. Is f continuous? Check if $\int_0^2 f(x)dx$ and $\lim_{n\to\infty}\int_0^2 f_n(x)dx$ are equal or not. (6)

8. a) Find the radius of convergence of the series $\sum a_n x^n$ where $a_n=\dfrac{n!}{n^n}$. (3)

b) $$f(x)=\begin{cases}\dfrac{x+2}{x^2-4}, & \text{when } x\neq -2\\ -1 & , \text{when } x=-2\end{cases}$$

Check whether f is uniformly continuous on $[-1,1]$ or not. (3)

c) Show that

$$e^{-x} < 1 - x + \frac{x^2}{2!}, x > 0. \quad (4)$$

9. a) Show that the sequence $\{f_n\}$ of functions, where

$$f_n(x) = \frac{n}{x+n}$$

is uniformly convergent in $[0,k]$, where $k > 0$. Show further that $\{f_n\}$ is not uniformly convergent in $[0,\infty[$. (6)

b) Evaluate $\int_0^1 x^2 dx$ using Riemann integration. (4)

10. a) Find the value/s of x for which the series

$$\sum \frac{1.3.5....(2n-1)}{2.4.6.....2n} \cdot \frac{x^n}{n}$$

is convergent. (4)

b) Give one example each for the following. Justify your choice of examples. (6)

i) A sequence which is divergent.
ii) A set which is neither open nor closed.
iii) A compact set.
iv) A set which has no limit point.