BMTC-131

ASSIGNMENT BOOKLET

CALCULUS

Valid from 1st Jan, 2025 to 31st Dec, 2025

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ……………………….….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: BMTC-131
Assignment Code: BMTC-131/TMA/2025
Maximum Marks: 100

1. Which of the following statements are true or false? Give reasons for your answer in the form of a short proof or a counter-example, whichever is appropriate. (10)

   a) The set $\{S \in \mathbf{R} : x^2 - 3x + 2 = 0\}$ is an infinite set.

   b) The greatest interger function is continuous on **R**.

   c) $\frac{d}{dx}\left[\int_3^{e^x} \ln t \; dt\right] = xe^x - \ln 3.$

   d) Every integrable function is monotonic.

   e) $a \oplus b = \sqrt{a+b}$ defines a binary operation on **Q**, the set of rational numbers.

2. a) Find the domain of the function f given by $f(x) = \sqrt{\frac{2-x}{x^2+1}}$. (5)

   b) The set **R** of real numbers with the usual addition $(+)$ and usual multiplication (.) is given. Define (*) on **R** as:

   $$a * b = \frac{a+b}{2}, \forall a, b \in \mathbf{R}.$$

   Is $(*)$ associative in **R**? Is (.) distributive (*) in **R**? Check. (5)

3. a) If $|z - 1 + 2i| = 4,$ show that the point $z + i$ describes a circle. Also draw this circle. (5)

   b) Express $\frac{x-1}{x^3 - x^2 - 2x}$ as a sum of partial fractions. (5)

4. a) Find the least value of $a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x,$ where $a > 0, b > 0.$ (5)

   b) Evaluate: (3)

   $$\int \frac{x^2 \cot^{-1}(x^3)}{1+x^6} dx$$

   c) For any two sets S and T, show that:

   $$S \cup T = (S - T) \cup (S \cap T) \cup (T - S).$$

   Depict this situation in the Venn diagram. (2)

5. a) Let f and g be two functions defined on **R** by: (5)

$$f(x) = x^3 - x^2 - 8x + 12$$

and $g(x) = \begin{cases} \dfrac{f(x)}{x+3}, & \text{when } x \neq -3 \\ \alpha \;, & \text{when } x = -3 \end{cases}$

i) Find the value of α for which f is continuous at $x = -3$.

ii) Find all the roots of $f(x) = 0$.

b) Find the area between the curve $y^2(4-x) = x^3$ and its asymptote parallel to y-axis. (5)

6. a) If the revenue function is given by $\dfrac{dR}{dx} = 15 + 2x - x^2$, x being the input, find the maximum revenue. Also find the revenue function R, if the initial revenue is 0. (5)

b) Trace the curve $y^2(x+1) = x^2(3-x)$, clearly stating all the properties used for tracing it. (5)

7. a) Find the length of the cycloid $x = \alpha(\theta - \sin\theta), y = \alpha(1-\cos\theta)$ and show that the line $\theta = \dfrac{2\pi}{3}$ divides it in the ratio 1 : 3. (5)

b) Find the condition for the curves, $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersecting orthogonally. (5)

8. a) If $y = e^m \sin^{-1} x$, then show that $(1-x^2)y_2 - xy_1 - m^2 y = 0$. Hence using Leibnitz's formula, find the value of $(1-x^2)y_{n+2} - (2n+1)xy_{n+1}$. (6)

b) Find the largest subset of **R** on which the function $f : \mathbf{R} \to \mathbf{R}$ defined as: (4)

$$f(x) = \begin{cases} 2x & , x > 5 \\ x+5, & 1 \le x \le 5 \\ |x| & , x < 1 \end{cases}$$

is continuous.

9. a) Solve the equation: (5)

$$x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$$

given that its roots are in G.P.

b) Evaluate: (5)

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

10. a) If $I_{m,n} = \int x^3 (\log x)^n dx$, show that:

$$(m+1) I_{m,n} = x^{m+1} (\log x)^n - n I_m , n-1.$$

Hence find the value of $\int x^4 (\log x)^3 dx$. (6)

b) Verigy Lagrange's mean value theorem for the function f defined by $f(x) = 2x^2 - 7x - 10$ over [2, 5]. (4)