BMTE-144

ASSIGNMENT BOOKLET

Bachelor of Science(General)/Bachelor of Arts(General) (BSCG/BAG)

NUMERICAL ANALYSIS

Valid from 1st January, 2024 to 31st December, 2024

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE : ………………………..….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: BMTE-144
Assignment Code: BMTE-144/TMA/2024
Maximum Marks: 100

1. a) Find the largest real root α of $f(x) = x^6 - x - 1 = 0$ lying between 1 and 2. Perform three iterations by
   i) bisection method
   ii) secant method $(x_0 = 2,\ x_1 = 1)$. (4)

   b) Find the number of positive and negative roots of the polynomial $P(x) = x^3 - 3x^2 + 4x - 5$. Find $P(2)$ and $P'(2)$ using synthetic division method. (3)

   c) Solve $x^3 - 9x + 1 = 0$ for the root lying between 2 and 4 by the method of false position. Perform two iterations. (3)

2. a) Using $x_0 = -2$ as an initial approximation find an approximation to one of the zeros of

$$p(x) = 2x^4 - 3x^2 + 3x - 4$$

   by using Birge-Vieta method. Perform two iterations. (4)

   b) Find by Newton's method the roots of the following equations correct to three places of decimals
   i) $x \log_{10} x = 4.772393$ near $x = 6$
   ii) $f(x) = x - 2\sin x$ near $x = 2$ (6)

3. a) The equation $x^2 + ax + b = 0$ has two real roots p and q such that $|p| < |q|$. If we use the fixed point iteration $x_{k+1} = \dfrac{-b}{x_k + a}$, to find a root then to which root does it converge? (3)

   b) Estimate the eigenvalues of the matrix

$$\begin{bmatrix} 1 & -2 & 3 \\ 6 & -13 & 18 \\ 4 & -10 & 14 \end{bmatrix}$$

   using the Gershgorin bounds. Draw a rough sketch of the region where the eigenvalues lie. (4)

   c) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$$

   using Gauss Jordan method. (3)

4. a) Solve the system of equations

$$0.6x + 0.8y + 0.1z = 1$$
$$1.1x + 0.4y + 0.3z = 0.2$$
$$x + y + 2z = 0.5$$

by LU decomposition method and find the inverse of the coefficient matrix (5)

b) For the linear system of equations

$$\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

set up the Gauss-Jacobi and Gauss-Seidal iteration schemes in matrix form. Also check the convergence of the two schemes. (5)

5. a) Find the dominant eigenvalue and the corresponding eigenvector for the matrix

$$A = \begin{bmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

using five iterations of the power method and taking $\boldsymbol{y}^{(0)} = [1 1 1]^T$ as the initial vector. (5)

b) Solve the system of equations

$$3x + 2y + 4z = 7$$
$$2x + y + z = 7$$
$$x + 3y + 5z = 2$$

with partial pivoting. Store the multipliers and also write the pivoting vectors. (5)

6. a) Determine the constants $\alpha,\ \beta,\ \gamma$ in the differentiation formula

$$\mathrm{y}'(\mathrm{x}_0) = \alpha\, \mathrm{y}(\mathrm{x}_0 - \mathrm{h}) + \beta\, \mathrm{y}(\mathrm{x}_0) + \gamma\, \mathrm{y}(\mathrm{x}_0 + \mathrm{h})$$

so that the method is of the highest possible order. Find the order and the error term of the method. (4)

b) The function $f(x) = \ln(1+x)$ is to be tabulated at equispaced points in the interval $[2,\ 3]$ using linear interpolation. Find the largest step size h that can be used so that the error $\leq 5 \times 10^{-4}$ in magnitude. (3)

c) Using finite differences, show that the data

x	-3	-2	-1	0	1	2	3
$f(x)$	13	7	3	1	1	3	7

represents a second degree polynomial. Obtain this polynomial using interpolation and find $f(2.5)$. (3)

7. a) Derive a suitable numerical differentiation formula of $0(h^2)$ to find $f''(2.4)$ with $h = 0.1$ given the table

x	0.1	1.2	2.4	3.9
$f(x)$	3.41	2.68	1.37	-1.48

(4)

b) The position $f(x)$ of a particle moving in a line at various times x_k is given in the following table. Estimate the velocity and acceleration of the particle at $x = 1.2$.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$f(x)$	2.72	3.32	4.06	4.96	6.05	7.39	9.02

(3)

c) Take 10 figure logarithm to bases 10 from $x = 300$ to $x = 310$ by unit increment. Calculate the first derivative of $\log_{10} x$ when $x = 310$. (3)

8. a) Show that $\sqrt{1+\mu^2\delta^2} = 1 + \dfrac{\delta^2}{2}$ where μ and δ are the average and central differences operators, respectively. (2)

b) A table of values is to be constructed for the function $f(x)$ given by $f(x) = \dfrac{1}{1+x}$ in the interval $[1, 4]$ with equal step length. Determine the spacing h such that quadratic interpolation gives result with accuracy 1×10^{-6}. (3)

c) Using the classical R-K method of $O(h^4)$ calculate approximate solution of the IVP, $y' = 1 - x + 4y,\ y(0) = 1$ at $x = 0.6$, taking $h = 0.1$ and 0.2. Use extrapolation technique to improve the accuracy. (5)

9. a) Compute the values of

$$1 = \int_0^1 \frac{dx}{1+x^2}$$

by using the trapezoidal rule with $h = 0.5,\ 0.25,\ 0.125$. Improve this value by using the Romberg's method. Compare your result with the true value. (5)

b) Use modified Euler's method to find the approximate solution of IVP

$$y' = 2xy,\ y(1) = 1 \text{ at } x = 1.5 \text{ with } h = 0.1$$

If the exact solution is $y(x) = e^{x^2-1}$, find the error. (3)

c) Show that $u_x = c_1 e^{\alpha x} + c_2 e^{-\alpha x}$ is a solution of the difference equation $u_{x+1} - 2u_x \cosh\alpha + u_{x-1} = 0$. (2)

10. a) Using the following table of values, find approximately by Simpson's rule, the arc length of the graph $y = \dfrac{1}{x}$ between the points $(1, 1)$ and $\left(5, \dfrac{1}{5}\right)$

x	1	2	3	4	5

$\sqrt{\dfrac{1+x^4}{x^4}}$	1.414	1.031	1.007	1.002	1.001

(3)

b) i) Calculate the third-degree Taylor polynomial about $x_0 = 0$ for $f(x) = (1+x)^{1/2}$.

ii) Use the polynomial in part (i) to approximate $\sqrt{1.1}$ and find a bound for the error involved.

iii) Use the polynomial in part (i) to approximate $\int_0^{0.1} (1+x)^{1/2}\,dx$. (7)