BMTC-133

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

(BSCG / BAG)
REAL ANALYSIS

Valid from 1st January, 2024 to 31st December, 2024

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C, covering all the blocks of the course. The total marks of the three parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….…

STUDY CENTRE: …………………………..….. **DATE:** ……………………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2024 to 31st Dec, 2024**. If you have failed in this assignment or fail to submit it by Dec, 2024, then you need to get the assignment for the year 2025, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTC-133
Assignment Code: BMTC-133/TMA/2024
Maximum Marks: 100

Part A (20 Marks)

1. Which of the following statements are true or false? Give reasons for your answers in the form of a short proof or counter-example, whichever is appropriate: (10)

   i) Every infinite set is an open set.

   ii) The negation of $p \wedge \sim q$ is $p \to q$.

   iii) -1 is a limit point of the interval $]-2,1]$.

   iv) The necessary condition for a function f to be integrable is that it is continuous.

   v) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x-2| + |3-x|$ is differentiable at $x = 5$.

2. Give an example for each of the following. (10)

   i) A set in $\mathbb{R}$ with a unique limit point.

   ii) A set in $\mathbb{R}$ whose all points except the one are its limit points.

   iii) A set having no limit point.

   iv) A set S with $S^\circ = \overline{S}$.

   v) A bijection from $\mathbb{N}_{\text{odd}}$ to $\mathbb{Z}$.

Part B (30 Marks)

3. a) Give an example of a divergent sequence which has two convergent subsequences. Justify your claim. (3)

   b) The product of two divergent sequences is divergent. True or false? Justify. (2)

   c) Let $(a_n)_{n \in \mathbb{N}}$ be any sequence. Show that $\lim_{n\to\infty} a_n = L$ iff for every $\varepsilon > 0$, there exists some $N \in \mathbb{N}$ such that $n \geq N$ implies $a_n \in N_\varepsilon(L)$. (3)

   d) Show that $\left(\dfrac{1}{n^2+n+1}\right)_{n\in\mathbb{N}}$ is a Cauchy sequence. (3)

   e) Evaluate
   $$\lim_{n\to\infty}\left[\frac{n}{1+n^2} + \frac{n}{4+n^2} + \frac{n}{9+n^2} + \cdots + \frac{n}{2n^2}\right].$$ (4)

4. a) Determine the points of discontinuity of the function f and the nature of discontinuity at each of those points:

$$f(x)=\begin{cases}-x^2, & \text{when } x\le 0\\ 4-5x, & \text{when } 0<x\le 1\\ 3x-4x^2, & \text{when } 1<x\le 2\\ -12x+2x, & \text{when } x>2\end{cases}$$

Also check whether the function f is derivable at $x=1$. (5)

b) Find the following limit.

$$\lim_{x\to 0}\frac{1-\cos x^2}{x^2\sin x^2}$$ (3)

c) Prove that a strictly decreasing function is always one-one. (2)

d) Determine the local minimum and local maximum values of the function f defined by $f(x)=3-5x^3+5x^4-x^5$. (5)

Part C (50 Marks)

5. a) Let $f:[0,1]\to\mathbb{R}$ be a function defined by $f(x)=x^m(1-x)^n$, where $m,n\in\mathbb{N}$. Find the values of m and n such that the Rolle's Theorem holds for the function f. (3)

b) Let f be a differentiable function on $[\alpha,\beta]$ and $x\in[\alpha,\beta]$. Show that, if $f'(x)=0$ and $f''(x)>0$, then f must have a local maximum at x. (3)

c) Suppose that $f:[0,2]\to\mathbb{R}$ is continuous on $[0,2]$ and differentiable on $]0,2[$, and that $f(0)=0, f(1)=1, f(2)=1$. (4)

(i) Show that there exists $c_1\in(0,1)$ such that $f'(c_1)=1$.

(ii) Show that there exists $c_2\in(1,2)$ such that $f'(c_2)=0$.

(iii) Show that there exists $c\in(0,2)$ such that $f'(c)=\frac{1}{3}$.

6. a) Test the following series for convergence. (6)

(i) $\sum_{n=1}^{\infty} n x^{n-1}, x>0$.

(ii) $\sum_{n=1}^{\infty}\left[\sqrt{n^4+9}-\sqrt{n^4-9}\right]$

b) Show that $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5}{7n+2}$ is conditionally convergent. (4)

7. a) Use Cauchy's Mean Value Theorem to prove that:

$$\frac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}=\tan\theta, 0<\alpha<\theta<\beta<\frac{\pi}{2} \qquad (5)$$

b) Using Weiestrass M-test, show that the following series converges uniformly. (5)

$$\sum_{n=1}^{\infty} n^3\ x^n, x\in\left[-\frac{1}{3},\frac{1}{3}\right].$$

8. a) Use the Fundamental Theorem of Integral Calculus to evaluate the integral

$$\int_0^1\left(2x\sin\frac{1}{x}-\cos\frac{1}{x}\right)dx. \qquad (5)$$

b) Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+7$ has an inverse by applying the inverse function theorem. Find its inverse also. (5)

9. a) Check whether the series $\sum_{n=1}^{\infty}\frac{n^2 x^5}{n^4+x^3}$, $x\in[0,\alpha]$ is uniformly convergent or not, where $\alpha\in\mathbb{R}^+$. (4)

b) Show that the series $\sum_{n=1}^{\infty}\frac{\sin n\theta}{n}$ does not converge uniformly on the interval $]0,2\pi[$. (4)

c) If the power series $\sum_{n=0}^{\infty}a_n x^n$ converges uniformly in $]\alpha,\beta[$, then so does $\sum_{n=0}^{\infty}a_n(-x)^n$. True or false? Justify. (2)