BMTC-132

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

(BSCG / BAG)
DIFFERENTIAL EQUATIONS

Valid from 1st Jan, 2024 to 31st Dec, 2024

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C, covering all the blocks of the course. The total marks of the three parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ……………………………

STUDY CENTRE: ……………………….….. **DATE:** ……………………….…………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2024 to 31st Dec, 2024**. If you have failed in this assignment or fail to submit it by Dec, 2024, then you need to get the assignment for the year 2025, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTC-132
Assignment Code: BMTC-132/TMA/2024
Maximum Marks: 100

1. State whether the following statements are true or false. Give a short proof or a counter-example in support of your answer: (10×2=20)

   a) A real-valued function of three variables which is continuous everywhere is differentiable.

   b) The function $f(x, y) = \ln\left(\frac{x+y}{x}\right)$ is not homogeneous function.

   c) The cylindrical coordinates of the point whose spherical coordinates is $\left(8, \frac{\pi}{6}, \frac{\pi}{2}\right)$ is $\left(8, \frac{\pi}{6}, 0\right)$.

   d) The unique solution $y(x)$ of an ordinary differential equation:

   $$\frac{dy}{dx} = \begin{cases} 0, & for\ x < 0 \\ 1, & for\ x \geq 0 \end{cases}$$

   exists $\forall x \in \mathbb{R}$.

   e) The differential equation:

   $$\left[1 + (y')^2\right]^{\frac{5}{3}} = y''$$

   is a second order differential equation of degree 3.

   f) Differential equation:

   $$\cos x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy^2 = 0$$

   in $]0, \pi[$ is a linear, homogeneous equation.

   g) The total differential equation corresponding to the family of surfaces $x^3 z + x^2 y = c$, where c is a parameter is $3x^2 dz + x(y\,dx + x\,dy) = 0$.

   h) Differential equation:

   $$5x^2 y^2 z^2 = 2px^2 y^2 + 5qx^2 y^3 + 2pz^2 + 9x^2 y^2$$

   is a semi-linear partial differential equation of first order.

   i) The differential equation:

$$v\frac{du}{dv} = e^{2v} + uv - u$$

has an integrating factor $v\,\exp(-v)$.

j) The simultaneous differential equation of simple harmonic motion of a particle in phase –plane is:

$$\frac{dx}{y} = \frac{dy}{-w^2 x} = dt \text{ with } y(x_0) = y_0.$$

2. a) Solve the differential equation: (4)

$$\frac{dy}{dx}\frac{-\tan y}{(1+x)} = (1+x)e^x \sec y$$

b) Show that the differential equation: (6)

$$(y^2 + yx)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$$

is integrable and find its integral.

3. a) Solve the differential equation: (4)

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\ln x$$

b) Using Charpit's method, find the complete integral of the differential equation: (6)

$$p(1+q^2) + (b-z)q = 0,$$

b being a constant.

4. a) Find all the first order partial derivatives of the following function: (4)

$$h(x, y, t) = e^{x-t}\cos(y+t)$$

What is the value of $\frac{\partial h}{\partial t}$ at $\left(0, \frac{\pi}{2}, 0\right)$?

b) Find the limit of: (3)

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

as $(x, y) \to (0,0)$ along:

i) $y = 3x$

ii) $y = 5x$

What can you conclude about $\lim_{(x,y)\to(0,0)} f(x,y)$? Justify your answer.

c) If $\cos\alpha, \cos\beta$ and $\cos\gamma$ are the direction cosines of a line, then show that: (3)

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$$

5. a) Solve the differential equation: (4)

$$2x\frac{dy}{dx} + y(6y^2 - x - 1) = 0$$

b) The rate of change of the price of a commodity is proportional to the difference between the demand D and the supply S. If $D = \alpha - bP$ and $S = c\sin\beta t$, where a, b, c and β are constants, determine $P(t)$. It is given that at $t = 0, P = P_0$. (6)

6. a) Find the differential equations of the space curve in which the two families of surfaces:

$$u = x^2 + y^2 + 3z = c_1$$

and $v = zx + x^2 + y^2 = c_2$

intersect. (4)

b) Transform the given equation to Clairaut's form and hence find its general solution: (6)

$$xy(y - px) = x - py$$

Also find its singular solution, if it exists.

7. a) Find the envelope and the characteristic curves of the family of curves: (5)

$$x^2(y - c)^2 + z^2 = c^2\cos^2\alpha,$$

c and α are constants.

b) Using Lagrange's method, solve the differential equation: (5)

$$(x^2 - y^2 - z^2)p + 2xq = 2xz$$

8. a) Using the method of variation of parameters, solve the differential equation: (5)

$$\frac{d^2y}{dx^2} + y = \sec^3 x$$

b) Solve the differential equation: (5)

$$(2xe^y y^4 + 2xy^3 y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$$

9. a) Show that the limit of the function $f(x, y)$ exists at the origin, where:

$$f(x,y)=\begin{cases} x\cos\dfrac{1}{y}+y\cos\dfrac{1}{x}, & (x,y)\neq(0,0) \\ 0 \qquad\qquad\qquad , & (x,y)=(0,0) \end{cases}$$

Do the repeated limits of $f(x,y)$ exist? Justify your answer. (5)

b) For the function: (5)

$$f(x,y)=x^3+xy-2y^2,$$

Find the polynomial given by:

$$f_{xx}(1,2)(x-2)^2+f_{xy}(1,2)(x-2)(y-1)+f_{yy}(1,2)(y-1)^2$$