

BMTC-131

ASSIGNMENT BOOKLET

CALCULUS

(Valid from 1st January, 2024 to 31st December, 2024)

**It is compulsory to submit the Assignment before filling in the
Term-End Examination Form.**



**School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068**

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:

NAME:

ADDRESS:

.....

.....

COURSE CODE:

COURSE TITLE:

ASSIGNMENT NO.:

STUDY CENTRE: **DATE:**

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**
We strongly suggest that you retain a copy of your answer sheets.
- 7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
- 8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done after studying all the blocks)

Course Code: BMTC-131
Assignment Code: BMTC-131/TMA/2024
Maximum Marks: 100

1. State whether the following statements are True or False? Justify your answers with the help of a short proof or a counter example: (10)
- a) The function f , defined by $f(x) = \cos x + \sin x$, is an odd function.
- b) $\frac{d}{dx} \left[\int_2^{e^x} \ln t \, dt \right] = x - \ln 2$.
- c) The function f , defined by $f(x) = |x - 2|$, is differentiable in $[0, 1]$.
- d) $y = x^2 - 3x^3$ has no points of inflection.
- e) $y = -x^2$ is increasing in $[-5, -3]$.
2. a) Find $\frac{dy}{dx}$, if $y = x \sin^{-1} x + \sqrt{1 - x^2}$. (3)
- b) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$. (5)
- c) Find $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^2 - 5x + 4}$. (2)
3. Trace the curve $y^2 = x^2(x + 1)$ by stating all the properties used to trace it. (10)
4. a) Find the length of the curve given by $x = t^2, y = 2t^2$ in $0 \leq t \leq 2$. (4)
- b) Find the angle between the curves $y^2 = ax$ and $ay^2 = x^3$ ($a > 0$), at the points of intersection other than the origin. (6)
5. a) Evaluate $\int \frac{x^2 dx}{(x-3)(x-5)(x-7)}$ (4)
- b) Use Simpson's method to approximate $\int_0^8 (x^2 - x + 3) dx$ with 8 sub-intervals. (3)
- c) Find the derivatives of $\ln(1 + x^2)$ w.r.t. $\tan^{-1} x$. (3)
6. a) The curve $ay^2 = x(x - a)^2, a > 0$ has a loop between $x = 0$ and $x = a$. Find the area of this loop. (4)

- b) Obtain the largest possible domain, and corresponding range, of the function f , defined by $f(x) = \frac{x-2}{3-x}$. (2)
- c) Expand e^{2x} in powers of $(x-1)$, up to four terms. (4)
7. a) Verify Rolle's theorem for the function f , defined by $f(x) = x(x-2)e^{-x}$, on the interval $[0, 2]$. (5)
- b) Is the function f , defined by
- $$f(x) = \frac{x^2 - 5x + 4}{x^2 - 16}, x \neq 4$$
- $f(4) = 0$, continuous at $x = 4$? Give reasons for your answer. (2)
- c) Evaluate $\int_0^1 x^2 e^{3x} dx$. (3)
8. a) Using Trapezoidal rule, calculate $\int_0^1 \frac{dx}{1+x^2}$ by dividing the interval $[0, 1]$ in 5 equal subintervals. Hence evaluate π . (4)
- b) Find $\frac{dy}{dx}$, if $y = x^{\sin x} + (\sin x)^x$. (3)
- c) Find the perimeter of the cardioid $r = a(1 - \cos \theta)$. (3)
9. a) If the first three non-zero terms of Maclaurin's series for $\sin x$ are used to approximate $\sin \pi/2$, show that the error is less than $1/50$. (4)
- b) Find the least value of $a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x$, where $a > 0, b > 0$. (4)
- c) Evaluate $\lim_{x \rightarrow 0} (1+x)^{1/x}$ (2)
10. a) Find the slope of the normal to the curve $y = x^3$ at $\left(\frac{1}{2}, \frac{1}{8}\right)$. (2)
- b) Find the points of inflexion of the curve $y = \frac{a^2 x}{x^2 + a^2}$. Also, show that they lie on a straight line. (5)
- c) Evaluate $\int e^x \cdot \frac{x^2 - x + 1}{(1+x^2)^{3/2}} dx$ (3)