BMTE-144

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

(BSCG / BAG)
NUMERICAL ANALYSIS

Valid from 1st January, 2023 to 31st Dec, 2023

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: ……………………………

COURSE TITLE: ……………………………

ASSIGNMENT NO.: ……………………………

STUDY CENTRE: …………………………… **DATE:** ……………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2022 to 31st Dec, 2022**. If you have failed in this assignment or fail to submit it by Dec, 2022, then you need to get the assignment for the year 2023, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTE-144
Assignment Code: BMTE-144/TMA/2023
Maximum Marks: 100

PART – A (40 marks)

1. a) Find the approximate root of the equation $2x^3 = 3x + 6$ using Newton-Raphson method. Perform only 3 iterations with $x_0 = 2$. (3)

   b) The roots of the quadratic equation $x^2 + ax + b = 0$ are given by α and β. Show that the iteration $x_{k+1} = \dfrac{-(ax_k + b)}{x_k}$ will converge near $x = \alpha$ when $|\alpha| > |\beta|$. (4)

   c) If $\delta^2 f(x_0) = C_1 h^2 f''(x_0) + C_2 h^4 f^{(4)}(x_0) + \cdots$, find the values of C_1 and C_2. (3)

2. a) The Gauss-Seidel method is used to solve the system of equations

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

   Determine the rate of convergence of the method. (5)

   b) Find the interpolating polynomial by Newton's divided difference formula for the following data: (3)

x	0	1	2	4
y	1	1	2	5

   c) Using synthetic division method, show that 2 is a simple root of the equation

$$p(x) = x^4 - 2x^3 + x^2 - x - 2 = 0.$$ (2)

3. a) Using Gauss-Jordan method, find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$ (5)

   b) Find the largest step length that can be used for constructing a table of values for the function

$$f(x) = \frac{4}{3}x^3 + 5\,ln\,x,\ 10 \le x \le 20,$$

   so that a quadratic interpolation can be used with an accuracy of 5×10^{-6}. (5)

4. a) Find the missing values in the following table: (5)

x	0	1	2	3	4	5
y	0	2	–	18	–	90

b) Using Classical Runge-Kutta fourth order method, find an approximate value of $y(1.2)$ for the IVP $\frac{dy}{dx} = xy,\ y(1) = 2$ with $h = 0.2$. (5)

PART – B (40 marks)

5. a) For the following data, use Gauss backward difference method to obtain the interpolating polynomial $f(x)$:

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

Hence, find the value of $f(0.45)$. (5)

b) The velocity of a vehicle beginning from rest is given in the following table for part of the first four. Using Simpson's $\frac{1}{3}$ rule, find the distance travelled by the vehicle in this hour:

t = time in min.	10	20	30	40	50	60
v = velocity in km/hr.	80	60	70	75	70	80

(5)

6. a) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ by using trapezoidal rule with $h = 0.5$ and $h = 0.25$. Use Romber's method to find the best value of π. (5)

b) Estimate the eigenvalues of the matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & -2 \end{bmatrix}$$

using the Gerschgorin bounds. (5)

7. a) Determine the largest eigenvalue in magnitude and the corresponding eigenvector of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ using the power method. Take $(1,\ 0,\ o)^T$ as the initial approximation and perform 4 iterations. (5)

b) The method

$$x_{n+1} = \frac{1}{9}\left[5x_n + \frac{5N}{x_n^2} - \frac{N^2}{x_n^5}\right],\ n = 0, 1, 2, \ldots$$

where N is a positive constant, converges to $N^{1/3}$. Find the rate of convergence of the method. (5)

8. a) Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ using Gauss-Jordan method. (4)

b) Divide the polynomial

$$x^5 - 6x^4 + 8x^3 + 8x^2 + 4x - 40$$

by $(x-3)$ by the synthetic division method and find the remainder. (2)

c) Determine a unique polynomial $f(x)$ of degree ≤ 3 such that $f(x_0) = 1,\ f'(x_0) = 2,$ $f(x_1) = 2,\ f'(x_1) = 3,$ where $x_1 - x_0 = h.$ (4)

PART – C (20 marks)

9. a) Obtain the interpolating polynomial in simplest form which fits the following data:

x	−1	0	1	2
f(x)	3	−4	5	−6

. (3)

b) Prove that $\mu^2 = 1 + \frac{\delta^2}{4}.$ (2)

c) Determine the order of convergence of the iterative method

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

for finding a simple root of the equation $f(x) = 0.$ (5)

10. a) Solve the initial value problem using Euler method

$$y' = \frac{1}{x^2 - 3y},\ y(3) = 2.$$

Find $y(3.1)$ taking $h = 0.1.$ (2)

b) Set up the Gauss-Seidel iteration scheme in matrix form for solving the system of equations

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}.$$

Show that the method is convergent and hence find its rate of convergence. (5)

c) Write the error in linear interpolation. Hence, show that

$$|\,\text{error}\,| \le \frac{h^2}{8} \max |\,f''(x)\,|$$

where $h = x_1 - x_0,\ x \in [x_0,\ x_1].$ (3)