







**PART – B (40 Marks)**  
**(Based on Blocks 3 and 4 of the course)**

5. a) If  $f(x) = \sin^{-1} x$ , show that  $(1 - x^2)f''(x) - xf'(x) = 0$ . Hence, prove that  $f^{(n+2)}(0) = n^2 f^{(n)}(0)$ . (4)
- b) Find the maximum height of the curve  $y = 4 \sin x - 3 \cos x$  above the x-axis. (3)
- c) Verify Rolle's theorem for  $f$  on  $[-1, 1]$  defined by  $f(x) = x^4 - 4x^2 + 7$ . (3)
6. a) Find the approximate value of  $\sqrt[3]{0.99}$  upto 4 places of decimal. (4)
- b) Differentiate  $y$  w.r.t.  $x$  in the following cases: (6)
- i)  $y = \sqrt{\sin(\sqrt{x})}$
- ii)  $y = e^{\sin x} + x \ln x$
7. Which of the following statements are true and which are false? Give reasons for your answers, in the form of a short proof or a counter example.
- i) The function  $f$  defined by  $f(x) = \tan(2x)$  is a periodic function with period  $\pi$ .
- ii) The function  $f: \mathbf{R} \rightarrow \mathbf{R}$ , defined by  $f(x) = 1 - |x|$  is differentiable at  $x=1$ .
- iii) The function  $f: [3,4] \rightarrow \mathbf{R}$  defined by  $f(x) = x^2 - x$  is monotonic in its domain.
- iv) Every continuous function is differentiable.
- v) Every curve over  $\mathbf{R}$  has a point of inflection. (10)
8. Trace the curve  $y = \frac{8}{4-x^2}$ , and state all the properties you use to trace it. (10)

**PART – C (25 Marks)**  
**(Based on Block 5 of the course)**

9. Integrate the following functions w.r.t.  $x$ : (9)
- i)  $\int \frac{x-1}{x^3-x^2-2x} dx$
- ii)  $\int_2^4 x \ln x dx$
- iii)  $\int \frac{(\cot^{-1} x)^4}{1+x^2} dx$
10. (a) If  $I_n = \int_0^\infty e^{-x} \sin^n(x) dx$ , prove that  $(1 + n^2)I_n = n(n - 1)I_{n-2}$  for  $n \geq 2$ . (5)

b) Find the length of the curve given by  $x = t^3, y = 2t^2$  in  $0 \leq t \leq 1$ . What is the slope of the curve at  $t = \frac{1}{2}$ . (5)

11. Which of the following statements are true? Give reasons for your answers, in the form of a short proof or a counter example. (6)

i)  $\frac{d}{dx} \left[ \int_{x^2}^{\frac{\pi}{2}} e^{2 \cos t} dt \right] = -2x e^{2(\cos x^2)}$

ii)  $|x|$  is integrable in  $[-2, 2]$ .

iii)  $\int_a^b x dx = \frac{b^2 - a^2}{2}$ .