ASSIGNMENT BOOKLET

PROBABILITY AND STATISTICS

Valid from 1st Jan, 2025 to 31st Dec, 2025

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THE PEOPLE'S
UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

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Instructions for Formatting Your Assignments

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Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

<div align="right">

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: …………………….…….. **DATE:** …………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment
(To be done after studying all the blocks)

1. Which of the following statements are *True* or *False*? Give short proof or counter example in your answer. (10)

    i)   If the correlation coefficient between X and Y is -0.8, then the correlation coefficient between $2X - 1$ and $-3Y - 1$ is -0.48.

    ii) If X and Y are independent binomial variates with parameters (n_1, p_1) and (n_2, p_2) respectively, then $X + Y$ has binomial distribution with parameters $(n_1 + n_2, p_1 + p_2)$.

    iii) The function defined as

    $$f(x) = \begin{cases} |x| & ; \quad -1 < x < 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

    is a probability density function.

    iv) For a normal distribution with mean μ and variance σ^2, the hypotheses

    $H_1 : \mu = \mu_0, \sigma^2 = 1$ and

    $H_2 : \mu = \mu_0, \sigma^2 \geq 1$ are simple hypotheses.

    v)   In a problem of testing of a simple hypothesis against a simple alternative, if the probability of type-I error is known to be 0.06, then the power of the test will be 0.94.

2. The mean I.Q. of a large number of children of age 14 was 100 and standard deviation 16. Assuming that the distribution was normal, find (10)

    i)   the percentage of children having I.Q. under 80.

    ii) the limits in which the I.Q. of the middle 40% of the children will lie.

    You may like to use the following values:

    $P(Z > 1.25) = 0.1056$
    $P(Z < -0.525) = 0.3$

3. 6 observations on (X, Y) yielded the following data: (10)

    $$\sum X_i = 30, \sum Y_i = 180, \sum X_i Y_i = 1000,$$
    $$\sum X_i^2 = 200, \sum Y_i^2 = 5642.$$

    i)   Determine the correlation coefficient between X and Y.
    ii) Given $X = 10$, what will be the predicted value of Y?
    iii) Given $Y = 15$, what will be the predicted value of X?

4. A die is thrown 60 times with the following results: (10)

Face of die	1	2	3	4	5	6
Frequency	8	7	12	8	14	11

Test that the die is unbiased at 5% level of significance. Given that at 5, 6 and 7 d.f. the value of χ^2 are 11.070, 15.592 and 14.067 respectively.

5. Consider the joint probability density function (10)

$$f(x, y) = y^2 e^{-y(x+1)}; x \geq 0, y \geq 0.$$

Are both x and y regressions linear? Give reasons for your answer.

6. a) The mean and standard deviation of a variable x are m and σ respectively. Obtain the mean and standard deviation of $\dfrac{(ax+b)}{c}$, where a, b and c are constants. (5)

   b) If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13,$ determine a lower bound for $P(-2 < X < 8)$. (5)

7. a) Let E_1, E_2, E_3 and E_4 be arbitrary events. Write the following events in set notations: (4)

   i) not more than one of E_1, E_2, E_3, E_4.

   ii) one and only one of E_1, E_2, E_3, E_4.

   iii) E_1 and at least one of E_2, E_3, E_4.

   iv) none of E_2, E_3 and E_4 using E_1.

   b) Let the probability density function of r.v. X be (6)

$$f(x) = \begin{cases} 1+x \ ; & -1 < x \leq 0 \\ 1-x \ ; & 0 < x < 1 \\ 0 \ ; & \text{otherwise} \end{cases}$$

   and if $u = X$ and $v = X^2$, find $Cov(u, v)$. Also check the independence of u and v.

8. a) For a mesokurtic distribution with standard deviation 5, find fourth central moment m_4. (4)

   b) The probability that a card will have a flat tyre while crossing a certain bridge is 0.00005. Find the probability that, among 10,000 cars crossing the bridge, (6)

   i) exactly two cars will have a flat tyre.
   ii) at most two cards will have a flat tyre.

9. a) Let X_1 be an observation from an exponential distribution with the p.d.f. (5)

$$f(x) = \frac{1}{\theta} e^{-x/\theta}; x > 0$$

Test the null hypothesis that the mean of the distribution is $\theta = 2$ against the alternative hypothesis that is $\theta = 5$. The null hypothesis is accepted if and only if the observed value of the random variable is less than 3. Find the probabilities of type-I and type-II errors.

b) The mean and standard deviation of 20 items is found to be 10 and 2 respectively. At the time of checking it was found that one item having value 8 was incorrect. Calculate the mean and standard deviation if the wrong item is omitted. (5)

10. a) Let X be a gamma variable with parameters α and λ, having $E(X) = 6$ and $Var(X) = 3$. Find α and λ. Also, find the m.g.f. of a gamma variable, and hence verify that mean of X is 6 and variance of X is 3 using m.g.f. (6)

b) For married couples living in a certain locality, the probability that the husband will vote in a school board election is 0.21, the probability that they both will vote is 0.15. What is the probability that (4)

i) at least one of them will vote?
ii) neither of them will vote?