MTE-09

ASSIGNMENT BOOKLET

REAL ANALYSIS

Valid from 1st January, 2025 to 31st December, 2025

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

Course Code: MTE-09
Assignment Code: MTE-09/TMA/2025
Maximum Marks: 100

1. Are the following statements true or false? Give reasons for tour answers. (10)

   a) -2 is a limit point of the interval $]-3, 2]$.

   b) The series $\frac{1}{2}-\frac{1}{6}+\frac{1}{10}-\frac{1}{4}+\cdots$ is divergent.

   c) The function, $f(x)=\sin^2 x$ is uniformly continuous in the interval $[0, \pi]$.

   d) Every continuous function is differentiable.

   e) The function f defined on $\mathbb{R}$ by

   $$f(x)=\begin{cases} 0, & x \text{ is rational} \\ 2, & x \text{ is irrational} \end{cases}$$

   Is integrable in the interval $[2, 3]$.

2) a) Prove that the union of two closed sets is a closed set. Give an example to show that union of an infinite number of closed sets need not be a closed set. (4)

   b) Examine the function $f: \mathbb{R} \to \mathbb{R}$ defined by

   $$f(x)=\begin{cases} \frac{1}{6}(x+1)^3 & x \neq 0 \\ \frac{5}{6} & x = 0 \end{cases}$$

   for continuity on $\mathbb{R}$. If it is not continuous at any point of $\mathbb{R}$, find the nature of discontinuity there. (4)

   c) Find $\lim\limits_{x \to 0} \frac{1-\cos^2}{x^2 \sin x^2}$. (2)

3) a) Using the principle of mathematical induction, prove that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number, $\forall n \in N$. (4)

   b) Show that there is no real number, k for which the equation, $x^4-3x^2+k=0$ has two distinct roots in the interval $[2, 3]$. (3)

c) Let $f:[-3,3] \to \mathbb{R}$ be defined by $f(x) = 5(x) + x^3$, where $[x]$ denotes the greatest integer $\le x$. Show that this function is integrable. (3)

4. a) Prove that the function f defined by

$$f(x) = \begin{cases} 2, & \text{if } x \text{ is irrational} \\ -2, & \text{if } x \text{ is rational} \end{cases}$$

is discontinuous, $\forall x \in \mathbb{R}$, using the sequential definition of continuity. (4)

b) Examine the convergence of the following series: (4)

i) $\frac{3 \times 4}{5^2} + \frac{5 \times 6}{7^2} + \frac{7 \times 8}{9^2} + \cdots$

ii) $1 + 4x + 4^2 x^2 + 4^3 x^3 + \cdots (x > 0)$

c) Prove that the set of integers is countable. (2)

5. a) Prove that (4)

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{2n-1}} + \frac{1}{\sqrt{4n-2^2}} + \frac{1}{\sqrt{6n-3^2}} + \cdots + \frac{1}{n} \right] = \frac{\pi}{2}$$

b) Prove that the sequence $\left\{ \frac{a_n}{n} \right\}$ is convergent where $\{a_n\}$ is a bounded sequence. (3)

c) Prove that continuous function of a continuous function is continuous. (3)

6. a) Examine the function, $f(x) = (x+1)^3 (x-3)^2$ for extreme values. (4)

b) Show that the series $\sum_{n=1}^{\infty} \frac{x}{1 + n^2 x^2}$ is uniformly convergent in $[\alpha, 1]$ for any $\alpha > 0$. (4)

c) Give an example of an infinite set with finite number of limit points, with proper justification. (2)

7. a) Show that (4)

i) $\lim_{x \to \infty} \left(\frac{x-3}{x-1} \right)^x = \frac{1}{e^2}$

ii) $\lim\limits_{x \to \frac{5}{3}} \frac{1}{(3x+5)^2} = \infty$

b) For the function, $f(x) = x^2 - 2$ defined over $[1,5]$, verify : $L(P, f) \leq U(-P, f)$ where P is the partition which divides $[1,5]$ into four equal intervals. (3)

c) Let $\{a_n\}$ be a sequence defined as $a_1 = 3, a_{n+1} = \frac{1}{5} a_n$ show that $\{a_n\}$ converges to zero. (3)

8. a) Using the sequential definition of the continuity, prove that the function f, defined by: (4)

$$f(x) = \begin{cases} 3, & \text{if } x \text{ is irrational} \\ -3, & \text{if } x \text{ is rational} \end{cases}$$

is discontinuous at each real number.

b) Show that on the curve, $y = 3x^2 - 7x + 6$, the chord joining the points whose abscissa are $x = 1$ and $x = 2$, is parallel to the tangent at the whose abscissa is $x = \frac{3}{2}$. (4)

c) Give an example of a series $\sum a_n$ such that $\sum a_n$ is not convergent but the sequence (a_n) converges to 0. (2)

9. a) Test the series: (3)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sin nx}{n\sqrt{n}}$$

for absolute and conditional convergence.

b) Check whether the function f given by: (4)

$$f(x) = (x-4)^3 (x+1)^2$$

has local maxima and local minima.

c) Check, whether the collection G, given by: (3)

$$G' = \left\{ \left] \frac{1}{n+2}, \frac{1}{n} \right[: n \in N \right\}$$

is an open cover of $]0,1[$.

10. a) State Bonnet's mean value theorem for integrals. Apply it to show that: (4)

$$\left| \int_{3}^{5} \frac{\cos x}{x} dx \right| \le \frac{2}{3}$$

b) Show that the sequence (a_n), where $a_n \frac{n}{n^2+4}$ is monotonic. Is (a_n) a Cauchy sequence? Justify your answer. (4)

c) Check whether the intervals $[2,5]$ and $[7,10]$ are equivalent or not. (2)