MTE-08

ASSIGNMENT BOOKLET

DIFFERENTIAL EQUATIONS

(Valid from 1st Jan, 2025 to 31st Dec, 2025)

It is compulsory to submit the assignment before filling in the exam form.

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(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet covering all the blocks of the course. The total marks of the three parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: …………………………….. **DATE:** ……………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is valid only upto December, 2025. If you have failed in this assignment or fail to submit it by December, 2025, then you need to get the assignment for the year 2026 and submit it as per the instructions given in the programme guide.
7) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment (MTE-08)

Course Code: MTE-08
Assignment Code: MTE-08/TMA/2025
Maximum Marks: 100

1. Classify the following statements as true or false. Give a short proof of a counter example in support of your answer.

   i) The solution of the differential equation $\frac{dy}{dx} = y$ with $y(0) = 0$ exists, but is not unique.

   ii) The differential equation representing all tangents $ty = x + t^2$ at the point $(t^2, 2t)$ to the parabola $y^2 = 4x$ is $x(y')^2 + yy' + 1 = 0$.

   iii) The p.d.e.
   $a u_{xx} + 2b\, u_{xy} + c\, u_{yy} = 0$
   where a, b, c are constants is irreducible when $b^2 - ac = 0$.

   iv) The functions $f_1(x) = \cos^2 x$, $f_2(x) = \sin^2 x$, $f_3(x) = \sec^2 x$ and $f_4(x) = \tan^2 x$ are linearly dependent on the interval $]-\pi/2, \pi/2[$.

   v) The solution of the second order partial differential equation $\frac{\partial^2 u}{\partial y \partial x} = x^3 - y$ involves two arbitrary constants. (10)

2. a) Using the method of variation of parameters, solve the equation

   $$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x,\ 0 \le x < \pi/2.$$ (4)

   b) A mass m, free to move along a line is attracted towards a given point on the line with a force proportional to its distance from the given point. If the mass starts from rest at a distance x_0 from the given point, show that the mass moves in a simple harmonic motion. (4)

   c) Show that, for the differential equation

   $$\frac{d^2 y}{dx^2} + a(x)\frac{dy}{dx} + b(x) y = 0$$

   e^{mx} is a particular integral if $m^2 + am + b = 0$. Hence find the value of m so that e^{mx} is a particular integral of the equation

   $$(x-2)\frac{d^2 y}{dx^2} - (4x-7)\frac{dy}{dx} + (4x-6)y = 0$$ (2)

3. a) Obtain the Riccati equation associated with the equation $y'' + w^2 y = 0$ and hence find its solution. (2)

   b) Solve the differential equation $\frac{d^2 y}{dx^2} = a + bx + cx^2$, given that $\frac{dy}{dx} = 0$ and $y = d$, when $x = 0$. (2)

c) Find the general solution of the DE

$$y^2 \ln y = x\,py + p^2$$

Does the equation has any singular solution? If yes, obtain it. (4)

d) Reduce the equation $x^2(y - px) = yp^2$ to clairaut's form and hence find its complete solution. (2)

4. a) If f and g are arbitrary functions of their respective arguments, show that $u = f(x - vt + i\alpha y) + g(x - vt + i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, where $\alpha^2 = 1 - \frac{v^2}{c^2}$. (2)

b) Solve the following differential equations:

i) $$\frac{dx}{y^2(x-y)} = \frac{dy}{x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$$ (3)

ii) $(yz + z^2)dx - xzdy + xydz = 0$ (2)

iii) $(z + z^3)\cos x\; dx - (z + z^3)dy + (1 - z^2)\;(y - \sin x)dz = 0$. (3)

5. a) If $y_1 = 2x + 2$ and $y_2 = -x^2/2$ are the solutions of the equation

$$y = xy' + (y')^2/2$$

then are the constant multiples $c_1 y_1$ and $c_2 y_2$, where c_1, c_2 are arbitrary, also the solutions of the given DE? Is the sum $y_1 + y_2$ a solution? Justify your answer. (2)

b) Find the orthogonal trajectories of the family of parabolas $x = cy^2$ and sketch their graph. (3)

c) Find the general solution of the differential equation

$$\frac{y^2}{2} + 2ye^t + (y + e^t)\frac{dy}{dt} = 0.$$ (5)

6. a) Solve the following boundary value problem

$$u_t = u_{xx},\ 0 < x < l,\ t > 0$$
$$u(0, t) = u(l, t) = 0$$
$$u(x, 0) = x(l - x),\ 0 \le x \le l.$$ (6)

b) Solve the PDE

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \ln x.$$ (4)

7. a) Solve: $\frac{d^2 y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$. (3)

b) Solve the IVP $x^2\frac{dy}{dx} = \cos x - 2xy,\ y(\pi) = 0,\ x > 0$. (3)

c) Solve: $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 4\cos\ln(1+x)$. (4)

8. a) Verify that the equation $2(y+z)dx - (x+z)dy + (2y - x + z)dz = 0$ is integrable and find its primitive. (3)

b) Find the differential equation of the family of surfaces $\phi(x+y+z,\ x^2+y^2-z^2)=0$. What is the order of this p.d.e? (3)

c) Find the integral surface of the linear p.d.e. $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$, which contains the straight line $x+y=0,\ z=1$. (4)

9. a) Find the complete integral of $p^2+q^2-2px-2qy+1=0$. (4)

b) Solve the differential equation $p^2+2py\cot x - y^2 = 0$, where $p=\dfrac{dy}{dx}$. (4)

c) Find the directional derivatives of $f(x,\ y,\ z) = x^2+2y^2+3z^2$ at $P_0(1,\ 1,\ 1)$ in the direction of $\boldsymbol{a} = \boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$. (2)

10. a) Find the deflection of the fixed end vibrating string of unit length corresponding to zero initial deflection and $u(x)$ given below as the initial velocity.

$$u(x) = \begin{cases} x, & 0 \le x < \dfrac{1}{2} \\ 1-x, & \dfrac{1}{2} \le x < 1 \end{cases}.$$ (4)

b) Using Jacobi's method find the complete integral of the equation $2u_1xz + 3u_2y^2 + u_2^2u_3 = 0$. (4)

c) Solve: $(D^2 - 2DD' + D'^2)z = 12xy$. (2)