MTE-14

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

MATHEMATICAL MODELLING

(Valid from 1st January, 2024 to 31st December, 2024)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE : …………………………..….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-14
Assignment Code: MTE-14/TMA/2024
Maximum Marks: 100

1. a) State two real-world problems where you think that mathematical modelling is the only approach to find the solution of the problem. Give 4 essentials for each of the problems. Why do you think that there is no other scientific alternative for the treatment of these problems. (6)

   b) Classify the following into linear and non-linear models, justifying your classification.

   i) Simple harmonic motion for small amplitude of oscillation.

   ii) Population growth model given by

   $$\frac{dN}{dt} = aN\,(B - kN),\ a,\ B,\ k \text{ are constants.}$$

   iii) Equation for velocity v of a particle at any time t, moving with a constant acceleration a, and initial velocity u.

   iv) Equation describing dynamic stability of market equilibrium price given by

   $$p_t = [1 + k\,(a - A)\,p_{t-1} + k\ (b - B)],\ a,\ b, A,\ B,\ k \text{ are constants}$$

   and p_t is the price in period t. (4)

2. a) Characterise the following as discrete or continuous giving reasons for your answers.

   i) Effects of radiation treatment on a tumour when applied for short period of time but at regular intervals.

   ii) Effects of chemotherapy drugs on a tumour when introduced into a patient for a given duration of time. (2)

   b) A particle of mass m moves on a straight line towards the centre of attraction, starting from rest at a distance a from the centre. Its velocity at a distance x from the centre varies as $\sqrt{\frac{a^3 - x^3}{x^3}}$. Find the law of force. (3)

   c) If a planet was suddenly stopped in its orbit, supposed circular, show that it will fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. (5)

3. a) A particle moving in S.H.M has got the velocities $8\text{cm}/\text{sec}$ and $6\text{cm}/\text{sec}$ when it is at distance 3cm and 4cm respectively from the centre of its motion. Determine the period and the amplitude of motion. (4)

   b) Consider the following system of equations

   $$\frac{dx}{dt} = y,\ \frac{dy}{dt} = -k^2 \sin x$$

   Find the nature of the critical point $(0,\ 0)$ of the corresponding linear system. (3)

   c) A string of length l is connected to a fixed point at one end and to a stick of mass m at the other. The stick is whirling in a circle at constant velocity v. Use dimensional analysis to find the equation of the force in the string. (3)

4. a) A parachutist, whose weight (actually mass) is 64 kg, drops from a helicopter 5000 m. above the ground. She falls towards the earth under the influence of gravity. Assume that the gravitational force is constant. Assume that the force due to air resistance is proportional to the velocity of the parachutist. The proportionality constant is $k_1 = 16\ \text{kg/sec}$ when the parachute is closed, and is $k_2 = 100\ \text{kg/sec}$ when it is open. If the parachute does not open until 1 minute after the parachutist leaves the helicopter, after how many seconds will she hit the ground? (8)

b) A projectile is fixed with a constant speed v at two different angles of projection α and β such that it gives the same range. Show that $\operatorname{cosec}\alpha = \sec\beta$. (2)

5. a) Consider a one-dimensional growth $c(x, t)$ of phytoplankton in a water mass. Formulate the model describing the dynamics of growth taking into account the following: D, its diffusion coefficient, r its rate of growth, R its mortality rate due to sinking. Fixing the area of interest as $0 \le x \le 2$ and the initial concentration of phytoplankton as $30\ \text{moles/cm}^3$, find the concentration distribution of phytoplankton in $0 \le x \le 2$ at any time t. (7)

b) When an aeroplane ascends from take-off to an altitude of 10 km, by how much does the gravitational attraction acting on it decrease? (3)

6. a) Consider the group of individuals born in a given year $(t = 0)$ and let $n(t)$ be the number of these individuals surviving t year later. Let $x(t)$ be the number of members of this group who have not had smallpox by year t and are therefore still susceptible. Let β be the rate at which susceptibles contract smallpox and let v be the rate at which people who contract smallpox die from the disease. Finally, let $\mu(t)$ be the death rate from all causes other than smallpox. If dx/dt and dn/dt are, respectively the rates at which the number of susceptibles and entire population decline due to contraction from smallpox and also due to death from all causes then

i) Formulate the above problem by writing equations for dx/dt and dn/dt.

ii) Taking $z = x/n$, show that z satisfies the initial value problem

$$\frac{dz}{dt} = -\beta z(1 - vz),\ z(0) = 1$$

iii) Find $z(t)$ at any time t.

iv) Bernoulli estimated that $v = \beta = \frac{1}{8}$. Using these values, determine the proportion of 20 years old who have not had smallpox. (10)

7. a) A monopolist sets a price 'p' per unit and the quantity demanded 'q' is given by the following relation:

$$q = 17 - p.$$

Let there be a fixed cost of Rs.9 and a marginal cost of Rs.1 per unit.

i) Write the profit function of monopolist.

ii) For maximum profit, find the number 'x' of units produced. Also find the maximum profit.

iii) A potential entrant enters into the business of the monopolist. He believes that the monopolist will go on making 'x' units. Write the profit function of the entrant.

iv) For maximum profit of entrant, find the number 'z' of units produced.

v) Find the maximum profit of the entrant. Explain whether he should enter into the business or not.

vi) Find the profit made by the monopolist after the entrant has entered into business. (10)

8. a) Consider the cubic total cost function

$$C = 0.06q^3 - 0.8q^2 + 13q + 10$$

Assume that the price of q is 15 per unit. Find the output which yields maximum profit. (3)

b) Apply dominance to find the optimum strategies of A and B from the pay-off matrix given below

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cc} \text{u} & \text{v} \end{array} \\ \text{Player A} \begin{array}{c} \text{x} \\ \text{y} \\ \text{z} \end{array} & \begin{bmatrix} 6(2) & 7(3) \\ 4(7) & 9(3) \\ 7(1) & 8(2) \end{bmatrix} \end{array}$$

(3)

c) Suppose that the previous forecast was 2090 and the actual value of the variable of interest for the period was 1985 and the oldest value of interest was 1955. Using the moving average technique based upon the most recent four observations find new forecast for the next period. (4)

9. a) Compare the phase diagrams of the systems:

i) $\dot{x} = y,\ \dot{y} = -x$

ii) $\dot{x} = xy,\ \dot{y} = -x^2$

by locating the equilibrium points and sketching the phase paths. (5)

b) Consider the epidemic model governed by the following equation

$$\frac{dx}{dt} = -\beta x\ (n + 1 - x)$$

with initial condition $x = n$ at $t = 0$. Here $x(t)$ is the number of susceptibles at time t, β is the contact rate. The population is assumed to be closed and homogeneously mixing. Let the contact rate be 0.002 and the number of susceptibles be 5000 initially

i) Find the density of the population when the rate of appearance of new cases is maximum.

ii) Find the time (in weeks) at which the rate of appearance of new cases is maximum.

iii) Obtain the maximum rate of appearance of new cases. (5)

10. a) For a given set of securities, all their portfolios lie on or within the boundary of the region shown in Fig.1.

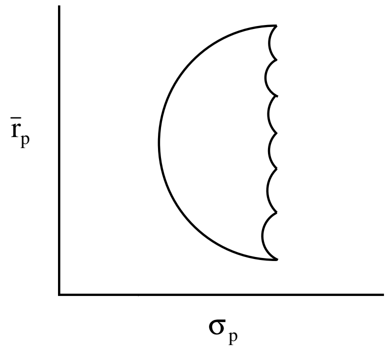

Fig.1

In the feasible region, find a portfolio which has maximum return. Also, find a portfolio in this region which has minimum risk. (3)

b) Explain the method of delineating the efficient frontier of a feasible region. (5)

c) Given all the portfolios of n securities what criterion would an investor use to select a good portfolio? (2)