MTE-07

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

ADVANCED CALCULUS

Valid from 1st January, 2024 to 31st December, 2024

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-07
Assignment Code: MTE-07/TMA/2024
Maximum Marks: 100

1. State whether the following statements are **true** or **false**. Justify your answer.

a) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$ is in $\left(\frac{0}{0}\right)$ form.

b) $f(x,y)=\dfrac{\sin\left(\frac{x^2y}{x^3+y^3}\right)}{\ln\left(\frac{x+y}{x}\right)}$ is a homogeneous function of degree 2.

c) Domain of $f(x,y)=\frac{xy}{x^4+y^4}$ is $\mathbf{R}^2$.

d) The function $f(x,y)=(x^3y+1, x^2+y^2)$ is locally invertible at $(1,2)$.

e) The function $f(x,y)=x^3+y^3$ is integrable on $[1,2]\times[1,3]$. (10)

2. a) Examine whether $\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x}+1}$ exists or not. (3)

b) If $f(x,y)=\begin{cases} x\sin\left(\frac{1}{y}\right)+y\sin\left(\frac{1}{x}\right), & xy\neq 0 \\ 0, & xy=0 \end{cases}$,

then show that the repeated limits of f do not exist. Examine the function f for simultaneous limit at the origin. (5)

c) Find the domain and range of the function f defined by $f(x,y)=\frac{3x^2y^2}{x^2+y^4}$. (2)

3. a) If $f(x,y)=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, $x\neq 0, y\neq 0$

$=0, \qquad x=0=y$

prove that $\frac{\partial^2 f}{\partial x\,\partial y}=\frac{x^2-y^2}{x^2+y^2}$. (5)

b) If $z=e^{xy^2}$, $x=t\cos t$, $y=t\sin t$, compute $\frac{dz}{dt}$ at $t=\frac{\pi}{2}$. (3)

c) Let $x = e_1 + e_2 - 2e_3, y = 2e_1 - e_2 + e_3$, where e_1, e_2, e_3 are unit vectors. Find $|x + 2y|$ and $|x + y|$. (2)

4. a) Check whether the function $f : \mathbf{R^2} \to \mathbf{R}$ defined by $f(x, y) = 2x^4 - 3x^2y + y^2$ has an extrema at $(0,0)$. (4)

b) Check whether there exists a continuously differentiable function g defined by $f(x, y) = 0$ in the neighbourhood of $x = 3$, such that $g(3) = \frac{1}{3}$. Find $g'(3)$, if it exists. (3)

c) Show that the functions $f(x, y) = \ln x - \ln y$ and $g(x, y) = \frac{x^2 + 2y^2}{2xy}$ are functionally dependent. (3)

5. a) Evaluate the integral of $f(x, y, z) = x + z - 3$ over the cylinder bounded by $x^2 + y^2 = 1, z = 0$ and $z = 1$. (3)

b) Evaluate the integral by converting to polar coordinates $\int_0^{\sqrt{3}} \int_y^{\sqrt{4-y^2}} \frac{dx\ dy}{4 + x^2 + y^2}$. (4)

c) Find two level curves of the function

$$f(x, y) = \frac{x + y}{x - y},\ x \neq y$$

and sketch them. (3)

6. a) If possible, find a function f such that $F = (4x^3 + 9x^2y^2,\ 6x^3y + 6y^5) = \nabla f$. (5)

b) Find the second Taylor's expansion of the function $f(x, y) = xy^2 + \cos xy$ about the point $\left(1, \frac{\pi}{2}\right)$. (3)

c) Find the values of a, b, c such that

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = \frac{3}{2}.$$ (2)

7. a) Show that

$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2}\, dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2}\, dx.$$ (4)

b) Show that the line integral

$$\int_{(-1,2)}^{(3,1)} (y^2 + 2xy)\, dx + (x^2 + 2xy)\, dy$$

is independent of path and evaluate it. (4)

c) Find the slopes of the tangents to the curves of intersections of the planes $x = 0,\ y = 2$ and the surface $z = x^3 + e^{yx}$ at the point $(0,2,1)$. (2)

8. a) Find the stationary points and the local extreme values of the function $f(x,y) = y^4 + xy^2 + x^2$ where $(x,y) \in \mathbf{R}^2$. (5)

b) Check whether the function

$$f(x,y) = \begin{cases} \dfrac{6xy}{\sqrt{x^2 + y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous at the origin. (5)

9. a) Find the mass of an object which is in the form of a cuboid $[0,1]\times[2,4]\times[1,3]$. The density at any point (x,y,z) on the cuboid is given by $\delta(x,y,z) = x^2 + y^2 + z^2$. (5)

b) Find the point on the ellipse $\dfrac{x^2}{4} + y^2 = 1$, that is nearest to the origin. (3)

c) Find fog and gof, if they exist, for the functions (2)

$f(t) = 4t,\ t \in \mathbf{R},\ g(x,y) = x + y,\ x, y \in \mathbf{R}$.

10. a) Let the function $f : \mathbf{R^2} \to \mathbf{R}$ be defined as (5)

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Show that

i) $f_x(0,y) = y$, for all y

ii) $f_x(x,0) = x$, for all x.

Hence, verify that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

b) Let $f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & \text{if } x^4 + y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$ (3)

Check whether $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists or not.

c) Prove that $\lim_{x \to 0} x \sin \frac{2}{x} = 0$. (2)