ASSIGNMENT BOOKLET

MTE-05

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

ANALYTICAL GEOMETRY

Valid from 1st January, 2024 to 31st December, 2024

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

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COURSE CODE **:** …….

COURSE TITLE **:** …….

ASSIGNMENT NO. **:** ……..…

STUDY CENTRE **:** ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. Answer sheets received after the due date shall not be accepted. We strongly suggest that you **retain a copy of your answer sheets**.
7) This assignment is valid only upto **December, 2024**. If you have failed in this assignment or fail to submit it by **December, 2024**, then you need to get the assignment for 2025 cycle and submit it as per the instructions given in that assignment booklet.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-05
Assignment Code: MTE-05/TMA/2024
Maximum Marks: 100

1. Check whether the following statements are **true** or **false**. Justify your answer with a short explanation or a counter example. (2 ×10 =20)

(i) The numbers $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{3}{4}$ are the direction cosines of a line.

(ii) The points $(1, 2), (7, 6)$ and $(4, 4)$ are collinear.

(iii) The conic $12x^2 + 12xy + 3y^2 + 2x + y = 0$ is degenerate.

(iv) Intersection of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{4} = 1$ and the plane $y = 5$ is a circle.

(v) The conicoid $3x^2 + y^2 + 2xy + x - y - z + 1 = 0$ is non-central.

(vi) The line $y = x$ is a tangent to the parabola $y^2 = cx, c > 0$.

(vii) The equation $2x^2 + y + z + 1 = 0$ represents a paraboloid.

(viii) Projection of a line segment on a line perpendicular to it is the length of the line segment.

(ix) The lines $x = -y, z = 2$ and $x = y, z = 0$ intersect each other.

(x) Every planar section of a cylinder is a circle.

2. (a) Trace the conic $x^2 - 2xy + y^2 - 3x + 2y + 3 = 0$. (4)

(b) Prove that the conic passing through the points of intersection of two rectangular hyperbolas is also a rectangular hyperbola. (3)

(c) Show that the line $x = y$ touches the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, if $f + g = 0$. (3)

3. (a) Let P be the midpoint of the line segment joining the points $A(a + b, b)$ and $B(a - b, a + b)$. Find the slope of the line passing through P and $Q\left(b, -\frac{a}{2}\right)$. Under what conditions on a and b, this line is parallel to the y-axis? (3)

(b) (i) Show that $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -4 & 7 & 1 \end{vmatrix} = 0$ represents the equation of a line passing through $(2, 3)$ and $(-4, 7)$. (2)

(ii) Prove that the equation of a line through (x_1, y_1) and (x_2, y_2) can be expressed in the form $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$. (3)

(c) Find the eccentricity, foci, centre and directrices of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$. Also give a rough sketch of it. (3)

(d) Prove that the length of the chord of a parabola which passes through the focus and which is inclined at 30° to the axis of the parabola is four times the length of the latus rectum. (4)

4. (a) Find the equations of the line through $(1,3,4)$ and parallel to the line joining the points $(-4, 5, 3)$ and $(8, 9, 7)$. (3)

(b) Find the equation of the plane which passes through the line of intersection of the planes $3x + 4y - 5z = 9$ and $2x + 6y + 6z = 7$ and which is perpendicular to the plane $3x + 2y - 5z + 6 = 0$. (3)

(c) Find the distance of the origin from the plane which passes through $(2, 1, 8), (1, 0, 2)$ and $(-3, 4, 6)$. (4)

5. (a) Show that the plane $2x + y + 2z = 0$ is a tangent plane to the sphere $x^2 + y^2 + z^2 - 2x + 2y - 2z + 2 = 0$. (3)

(b) Find the equation of the sphere touching the plane $8x + 5y + 3z + 1 = 0$ at $(3, -1, -1)$ and cutting the sphere $x^2 + y^2 + z^2 - 2x + y - z - 6 = 0$ orthogonally. (4)

(c) Find the angle between the lines of intersection of the cone $4x^2 + y^2 + 4z^2 + 4yz + 2zx = 0$ and the plane $x + 2y + 3z = 0$. (5)

(d) Find the equation of the cylinder with base $x^2 + y^2 + z^2 - 3x - 6z + 9 = 0, x - 2y + 2z - 6 = 0$. (3)

6. (a) Show that the perpendiculars drawn from the origin to tangent planes to the cone $x^2 - y^2 + 5z^2 + 4xy = 0$ lie on the cone $x^2 - y^2 + z^2 + 4xy = 0$. (3)

(b) Transform the equation $x^2 + 2y^2 - 6z^2 - 2x - 8y + 3 = 0$ by shifting the origin to $(1, 2, 0)$ without changing the directions of the coordinate axes. What object does this new equation represent? Give a rough sketch of it. (4)

(c) Show that the conicoid $2x^2 + 2y^2 + xy - yz + zx + 2x - y + 5z + 1 = 0$ is central. Hence find its centre. (3)

7. (a) Examine which of the following conicoids are central and which are non-central. Also determine which of the central conicoids have centre at the origin. (4)

(i) $x^2 + y^2 + z^2 + 4x + 3y - z = 0$

(ii) $2x^2 - y^2 - z^2 + xy + yz - zx = 1$

(iii) $x^2 + y^2 - z^2 - 2xy - 3yz - 6zx + x - 2y + 5z + 4 = 0$

(b) Find the transformation of the equation $12x^2 - 2y^2 + z^2 = 2xy$ if the origin is kept fixed and the axes are rotated in such a way that the direction ratios of the new axes are $1, -3, 0; 3, 1, 0; 0, 0, 1$. (4)

(c) Find the projection of the line segment joining the points $(1, -1, 6)$ and $(4, 3, 2)$ on the line $\frac{x-4}{3} = -y = \frac{z}{5}$. (2)

8. (a) Identify and trace the conicoid $y^2 + 3z^2 = x$. Describe its sections by the planes $y = 0$ and $z = 0$. (5)

(b) Find the equation of tangent plane to the conicoid $x^2 + 3y^2 = 4z$ at $(2, -4, 13)$. Represent the tangent plane geometrically. (5)