## ASSIGNMENT BOOKLET

## Bachelor's Degree Programme

 (B.Sc./B.A./B.Com.)
## OPERATIONS RESEARCH

(Valid from $1^{\text {st }}$ January, 2024 to $31{ }^{\text {st }}$ December, 2024)

It is compulsory to submit the Assignment before filling in the Term-End Examination Form.
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THE PEOPLE'S UNIVERSITY

School of Sciences
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Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, which would consist of one tutor-marked assignment for this course. The assignment is in this booklet.

## Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. $\qquad$

NAME: $\qquad$

ADDRESS: $\qquad$

## COURSE CODE:

COURSE TITLE:
ASSIGNMENT NO.: $\qquad$
STUDY CENTRE:
DATE: $\qquad$

## PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. Answer sheets received after the due date shall not be accepted.

We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.
8) You cannot fill the Exam Form for this course till you have submitted this assignment. So solve it and submit it to your study centre at the earliest.

We wish you good luck.

## Assignment (To be done after studying all the blocks)

1. Which of the following statements are true? Give reasons for your answers.
a) The optimal solution of any integer linear programming problem can be obtained by rounding off the optimal solution of its LP relaxation.
b) In an optimal solution $\left(x_{1}^{*} x_{2}^{*}\right)$ of the LPP

Max. $4 x_{1}+3 x_{2}$
s.t.

$$
\begin{aligned}
& x_{1}+x_{2}=2 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

$x_{1}^{*}, x_{2}^{*}$ both cannot be positive.
c) An assignment problem can be considered as a special case of transportation problem.
d) In the inventory model with finite replenishment rate, if the replenishment rate is equal to consumption rate, the holding cost is 0 .
e) For a Queuing $\operatorname{Model}(\mathrm{M} / \mathrm{M} / 1):(\mathrm{GD} / \infty / \infty)$ with one server, if the service rate $\mu$ increases, the expected number of customers in the system decreases.
2. a) A sugar manufacture has two production processes. In one hour, Process I makes 100 kg of Grade I (high quality) sugar and also produces 140 kg of Grade II sugar as a byproduct. Process II makes in one hour 60 kg of Grade I (high quality) sugar and also produces as a by-product 40 kg of Grade II sugar. The manufacture is confident that during the festival season all the sugar that is made can be sold. He has committed to selling at least 6000 kg of Grade I and 5600 kg of Grade II for the season. The revenue earned by selling one kilogram of Grade I sugar is ₹ 4 (irrespective of the process used) and the revenue earned by selling one kilogram of Grade II sugar is ₹ 2 (irrespective of the process used). Formulate the problem of maximizing the total revenue earned as an LPP. Solve the problem by graphical-method.
b) Customers come to a coffee shop at the average rate of 32 per day ( 8 hours a day) in Poisson pattern. The waiter employed to serve the customers has left the job. The owner of the shop wants to hire a new the job. The owner of the shop wants to hire a new waiter. Two applicants, Abdul and Raju, have applied for the post. The service times of Abdul and Raju are exponentially distributed with mean 12 minutes and 10 minutes, respectively. Abdul and Raju demand salaries of ₹ 135 and ₹ 165 per day, respectively. If no service is available, the average loss to the owner is ₹ 400 per day. Who among the two should be selected for the job?
3. a) A manager wants to the appoint 4 sales-persons to 4 different cities. If the expected profit when different persons are appointed to different cities is a given in the table below, find the assignment that will maximize the profit:
Sales - persons
Cities

|  | I |  | II | III |
| :---: | :---: | :---: | :---: | :---: |
| IV |  |  |  |  |
|  | 7 | 7 | 5 | 3 |
| B | 5 | 3 | 2 | 9 |
|  | 3 | 4 | 9 | 5 |
|  | 6 | 3 | 1 | 8 |
|  |  |  |  |  |

b) An oil engine manufacture purchase lubricants at the rate of ₹ 50 per unit from a vendor. The requirement of these lubricants is 1800 units per year. The cost of placing an order is ₹ 40 and inventory carrying cost per rupee per year is only 20 paise. Find the Economic Order Quantity (EOQ). Also find the cycle time.
4. a) The following is the optimal table of a maximising LPP where $x_{3}, x_{4}$ and $x_{4}$ are slack variables.

| $p_{B}$ | Basic <br> Variables | 3 | 5 | 0 | 0 | 0 | Solution |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ |  |  |
| 0 | $x_{1}$ | 1 | 0 | $\frac{1}{3}$ | 0 | $-\frac{2}{3}$ | 2 |
| 5 | $x_{2}$ | 0 | 0 | $-\frac{2}{3}$ | 1 | $\frac{4}{3}$ | 0 |
|  |  | 0 | 0 | -1 | 0 | -3 | 36 |

Suppose a new constant $2 x_{1}+x_{2} \leq 8$ is added to the LPP. Find the new optimal solution of the resulting LPP.
b) In a factory there are 6 jobs to be processed on two Machines A and B. The processing times are given in the table below. The jobs are first processed on Machine A and then on Machine B. Find the optimal job sequence and the minimum elapsed time.

| Job: | $J_{1}$ | $J_{2}$ | $J_{3}$ | $J_{4}$ | $J_{5}$ | $J_{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Machine A: | 1 | 3 | 8 | 5 | 6 | 3 |
| Machine B: | 5 | 6 | 3 | 2 | 2 | 10 |

5. a) For the following transportation problem, find an initial basic feasible solution by North-West Corner method.

| Factory | $W_{1}$ | $W_{2}$ | $W_{2}$ | Availability |
| :---: | :---: | :---: | :---: | :---: |
| $F_{1}$ | 16 | 20 | 12 | 200 |
| $F_{2}$ | 14 | 8 | 18 | 160 |
| $F_{3}$ | 26 | 24 | 16 | 90 |
| Demand | 180 | 120 | 150 |  |

Starting from the solution obtained by North-West Corner method, find an optimal solution and the optimal transportation cost.
b) A petrol station has a single pump and space for not more than 3 cars (2 waiting, 1 being served). Cars arrive according to a Poisson distribution at an average according to a Poisson distribution at an average rate of 2 per minute. The service time has an exponential distribution with an average rate of 4 per minute. Answer the following questions:
i) Find the probability that an arriving car doesn't have to wait.
ii) Calculate the expected waiting time until a car is served and leaves the petrol station.
6. a) A small project is composed of 8 activities where time estimates are listed in the table below:

| Activity | Estimated duration (in days) |  |  |
| :--- | :--- | :--- | :--- |
|  | Optimistic | Most likely | Pessimistic |
| A (1,2) | 28 | 32 | 36 |
| B (1,3) | 22 | 28 | 32 |
| C (2, 6) | 26 | 36 | 46 |
| D (3, 4) | 14 | 16 | 18 |
| E (3, 5) | 32 | 32 | 32 |
| F (3, 6) | 40 | 52 | 74 |
| G (4, 5) | 12 | 16 | 24 |
| H (5, 6) | 16 | 20 | 26 |

Draw the project network diagram. Using the PERT find the expected time and variance for each activity.
b) The time taken by a TV repair person to repair a TV set is exponentially distributed with mean 30 minutes. She repairs the sets in the order in which they come in. The arrival rate of the sets is approximately Poisson with an average rate of 10 per 8 hours day. Answer the following questions:
i) What is the repair person's expected idle time each day?
ii) How many jobs are ahead of the arriving set just brought in?
iii) What is probability that there are 2 or more sets in the system?
7. a) Find the shortest route using Bellman's principle.

b) Solve the following LPP by Simplex method:

Max. $x_{1}+7 x_{2}$
such that

$$
\begin{aligned}
3 x_{1}+5 x_{2} & \leq 15 \\
5 x_{1}+2 x_{2} & \leq 10 \\
x_{1}, x_{2} & \geq 0
\end{aligned}
$$

From the optimal table of the solution to the problem find the optimal solution of the dual of the problem. Verify complementary slackness property for the primal-dual pair.
8. a) Use the dual simplex method to solve the following L.P.P.
$\operatorname{Max} Z=-2 x_{1}-x_{3}$
Subject to
$x_{1}+x_{2}-x_{3} \geq 5$
$x_{1}-2 x_{2}+4 x_{3} \geq 8$
$x_{1}, x_{2}, x_{3} \geq 0$.
b) Solve the following cost minimizing assignment problem.

|  | $I$ | $I I$ | $I I I$ | $I V$ | $V$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 9 | 2 | 7 | 1 |
| $B$ | 6 | 8 | 7 | 6 | 1 |
| $C$ | 4 | 6 | 5 | 3 | 1 |
| $D$ | 4 | 2 | 7 | 3 | 1 |
| $E$ | 5 | 3 | 9 | 5 | 1 |
|  |  |  |  |  |  |

9. a) An investment company wants to study the investment proposals based on the profit factor. While analyzing a new investment proposal, the company estimated the probability distribution for the profit as follows:

| Profit (in <br> thousands) | 3 | 5 | 7 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.1 | 0.2 | 0.4 | 0.2 | 0.1 |

Using the random numbers:
19, 7, 90, 2, 57, 28
Simulate the profit of the company for six trials.
b) The production department for a company requires $3,600 \mathrm{~kg}$ of raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is ₹ 36 and the cost of carrying inventory is 25 percent of the investment in the inventories. The price of the raw material is ₹ 10 per kg.

Determine the following:
i) Economic order quantity
ii) Optimal order cycle time and
iii) Minimum yearly inventory cost.
10. a) Listed in the table below are the activities and sequencing requirements necessary for the completion of a project.

| Activity | Predecessor | Duration in <br> weeks |
| :---: | :---: | :---: |
| A | - | 6 |
| B | A | 24 |
| C | A | 6 |
| D | A | 12 |
| E | A | 9 |
| F | C, D, E | 18 |
| G | B, F | 12 |
| H | G | 24 |

i) Draw a net work diagram for the project.
ii) Find the critical path and the duration for the completion of the project.
b) Write the dual of the following LP problem:

Min. $Z=3 x_{1}-2 x_{2}+4 x_{3}$
Subject to

$$
\begin{aligned}
& 3 x_{1}+5 x_{2}+4 x_{3} \geq 7 \\
& 6 x_{1}+x_{2}+3 x_{3}=4 \\
& 7 x_{1}-2 x_{2}-x_{3} \leq 10 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{aligned}
$$

