MTE-09

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

REAL ANALYSIS

Valid from 1st January 2023 to 31st December 2023

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….…

STUDY CENTRE: …………………………..….. **DATE:** ……………………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2023. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-09
Assignment Code: MTE-09/TMA/2023
Maximum Marks: 100

1. Are the following statements **true** or **false?** Give reasons for your answer.

a) $\frac{1}{2}$ is a limit of the interval $]-2.5\ ,1.5\ [$

b) Every function differentiable on $[a,\ b]$ is bounded on $[a,\ b]$

c) The function f defined by $f(x)=\left|x-\frac{5}{2}\right|, x\in \mathbf{R}$ has a local maxima of $x=\frac{5}{2}$.

d) If $\lim\limits_{n\to\infty} u_n = 0$, then the series $\sum\limits_{n=1}^{\infty} u_n$ is convergent.

e) A Riemann integrable function is not necessarily differentiable. (10)

2. a) If $a\in \mathbf{R}$ is such that $o \le a \le \varepsilon\ \forall\ \ \varepsilon > 0$, then show that $a = 0$. (2)

b) Using the principle of mathematical induction, show that

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ (3)

c) Show that the union of a countable collection of open sets is open. (3)

d) Check whether the set of integers is compact or not. (2)

3. a) Check whether the following functions are continuous or not at $x = 0$. Also, find the nature of discontinuity at that point, if it exists.

(i) $$f(x)=\begin{cases}\dfrac{\sqrt{2-x}-\sqrt{2+x}}{x} & , \ x\neq 0\\[2ex] \dfrac{1}{\sqrt{2}} & , \ x=0\end{cases}$$

(ii) $$f(x)=\begin{cases} x^2+\dfrac{1}{3} & , \ x\le 0\\[2ex] -(x^3+\dfrac{1}{3}) & , \ x>0\end{cases}$$ (4)

b) Evaluate the following limit if it exists:

$$\lim_{x\to 0}\frac{4x^3}{\tan^3 x+\tan x - x}$$ (2)

c) Show that the function f given by

$$f(x)=\frac{1}{(x+2)^3}, \forall x \in]-2, 2[$$

is continuous but not bounded in $]-2, 2[$. (4)

4. a) For the following sequences, find two subsequences which are convergent:

(i) $a_n = n[1+(-1)^n]$.

(ii) $a_n = \sin\left(\frac{n\pi}{3}\right)$. (3)

b) Check whether the following sequences $\{s_n\}$ are Cauchy, where

(i) $s_n = 1+2+3+...+n$

(ii) $s_n = \frac{4n^3+3n}{3n^3+n^2}$ (3)

c) Give an example of an infinite set with finite number of limit points, giving justification. (2)

d) Evaluate: $\lim\limits_{x\to\infty}\left(\sqrt{2x^2+3x-2}-\sqrt{2x^2-3x+2}\right)$ (2)

5. a) Test for convergence the following series:

(i) $\sum\limits_{n=1}^{\infty} \frac{7^n}{(3n+1)!}$

(ii) $\sum\limits_{n=1}^{\infty} \frac{1}{n\sqrt{\log n}}$ (4)

b) Test the following series for absolute and conditional convergence:

(i) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{5}{3n+1}$

(ii) $\sum\limits_{n=1}^{\infty} \frac{\sin nx}{n^3}$ (4)

c) Show that the set $B=\{x \mid x^2 > 2\}$ is non-empty and bounded below. Is it bounded above? Justify. (2)

6. a) Show that the function f defined on **R** by

$$f(x)=\begin{cases} 3x^2\cos\frac{1}{2x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is derivable on **R** but f' is not continuous at x=0. (3)

b) Find the least and the greatest values of the function f defined by

$$f(x) = 3x^4 - 4x^3 + 6x^2 + 36x - 5$$

on the interval $[0, 2]$. (4)

c) Using Taylor's Theorem, prove that

$$\cos x \le 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ \forall\ x \in \mathbf{R}.$$ (3)

7. a) Test the following series for convergence:

$$\frac{2.4}{3.5} + \frac{2.4.6}{3.5.7}x + \frac{2.4.6.8}{3.5.7.9}x^2 + \ldots (x > 0).$$ (4)

b) Show that the function $f : [0,\ 1] \to \mathbf{R}$ defined by

$$f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ 2 & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable. (3)

c) Evaluate the limit as $n \to \infty$ of the sum

$$\frac{1}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots + \sin\frac{2n\pi}{n}\right]$$ (3)

8. a) Compute the Riemann integral of the function $f(x) = |x|$ on the interval $[-1, 1]$. (4)

b) Suppose that f is a non-negative continuous function on $[a, b]$ and $\int_a^b f(x)\ dx = 0$ Prove that $f(x) = 0 \forall x \in [a, b]$. (4)

c) Check whether the function $f(x) = [x] + e^x$ is integrable in $[0, 3]$. (2)

9. a) Verify the second mean value theorem of integrability for the functions f and g defined on $[1, 2]$ by $f(x) = 3x$ and $g(x) = 5x$. (4)

b) Show that the sequence (f_n) where $f_n(x) = \dfrac{x}{1 + 2nx^2}, x \in [1, \infty[$ is uniformly convergent in $[1, \infty[$. (3)

c) Verify Inverse function theorem for finding the derivative at a point y_0 of the domain of the inverse function of the function $f(x) = \cos x,\ x \in [0, \pi]$. Hence, find the derivative of the inverse function at y_0. (3)

10. a) Find the upper and lower integrals of the function f defined by

$$f(x) = \frac{7}{2} - 2x, \ \forall \ x \in [1,3].$$

Is f integrable over the interval $[1,3]$? Justify. (5)

b) Check whether the function $f(x) = \sin \frac{1}{x} (x \neq 0)$ is uniformly continuous in the interval $]0,2[$. Is it continuous? Justify. (3)

c) Find the value of $a \in \mathbf{R}$ for which

$$\lim_{x \to \infty} \frac{(3x+4)(x-1)(2x+1)}{ax^3 + x - 4} \text{ exists.}$$ (2)