MTE-11

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

PROBABILITY AND STATISTICS

(Valid from 1st January, 2023 to 31st December, 2023)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

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COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

Course Code: MTE-11
Assignment Code: MTE-11/TMA/2023
Maximum Marks: 100

1. Which of the following statements are true or false? Give reasons for your answer. (10)

   a) There exists a random variable X

   $$P[\mu - 2\sigma \le X \le \mu + 2\sigma] = 0.5.$$

   b) If for two events A and B, it is know that $P(A) = 0$ and $P(B) > 0$, then $P(A \mid B) = 0$.

   c) In a relative frequency histogram the are of each rectangle represents the relative frequency of that class.

   d) The variance of a binomial distribution with parameter (n, p), cannot exceed $n/4$.

   e) A maximum likelihood estimator is always an unbiased estimator.

2. a) The following data relate to the age of a group of Government employees. Calculate the mean and standard deviation: (4)

Age (in years)	No. of employees
50-55	25
45-50	30
40-45	40
35-40	45
30-35	80
25-30	110
20-25	170

   b) Records taken of the number of male and female births in 800 families having four children are given below: (6)

Number of Births		Frequency
Male	Female	
0	4	32
1	3	178
2	2	290
3	1	236
4	0	64

Test whether the data are consistent with the hypothesis that the binomial law holds for the chance of a male birth is equal to that of a female birth at the level of 5% significance.
(You may use the following values: $\chi^2_{4,\,0.05} = 9.488$, $\chi^2_{5,\,0.05} = 11.05$, $\chi^2_{6,\,0.05} = 12.59$)

3. a) Given below is some information about advertising and sales.

	Advertisement Expenditure (X) (Rs. Lakhs)	Sales (Y) (Rs. Lakhs)
Mean	10	90
S.D.	3	12

Correlation coefficient $= 0.8$.

i) Calculate the two regression lines.

ii) Find the likely sales when advertisement expenditure is Rs. 15 lakhs.

iii) What should the advertisement expenditure be if the company wants to attain a sales target of Rs. 120 lakhs? (3)

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with density function

$$f(x, \theta) = \theta^{-1} e^{-x/\theta}, \quad \theta > 0 \text{ if } x > 0$$
$$= \theta, \qquad \text{otherwise}$$

Calculate the mean and variance of the given distribution. Also show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is unbiased for θ and $\overline{X}$ has variance θ^2/n. (7)

4. a) Find the values of the third quartile and the third decile from the following data: (4)

Marks	**No. of students**
More than 70	7
More than 60	18
More than 50	40
More than 40	40
More than 30	63
More than 20	65

b) If a machine is correctly set up it will produce 90% acceptable items. If it is incorrectly set up it will produce 40% acceptable items. Experience shows that 80% of the set ups are correctly done.

i) If after a certain set up, out of the first two items produced the first is found to be acceptable and the second unacceptable, find the probability that the machine is correctly set up.

ii) If after a certain set up the machine produces two acceptable items as the first two pieces, find the probability that the machine is correctly set up. (6)

5. a) There are two coins – one unbiased with $P(H) = \frac{1}{2}$, the other biased with $P(H) = \frac{1}{3}$. One of these coins is selected and tossed 4 times. If the head comes up at least twice, the coin is assumed to be unbiased. Find the level of significance and power of the test. (4)

b) The first three moments of a distribution about the value 2 of a variable 1, 16 and -40. Show that the mean is 3, the variance is 15 and $\mu_3 = -86$. (4)

c) Is the following function $f(x) = 6x(1-x),\ 0 \le x \le 1$ a p.d.f. of a continuous random variable X? Give reasons for your answer. (2)

6. a) If a random variable X follows the Poisson distribution such that

$P(X = 1) = P(X = 2)$, find

i) the mean of the distribution,

ii) $P(X = 0)$,

iii) standard deviation of the distribution. (4)

b) Let the density function of a random variable X be

$$f(x) = \begin{cases} x/2, & 0 < x < 2 \\ x, & \text{otherwise} \end{cases}$$

Find the density function of $Y = 4 - X^3$. (3)

c) Determine the value of c so that the following function represents the joint p.m.f. of the random variables X and Y.

$f(x, y) = c(x^2 + y^2);\ x = -1, 1;\ y = -2, 2.$ (3)

7. a) Let $(X_1, X_2, \ldots, X_n)$ be a random sample from a distribution with density function

$$f(x, \alpha) = \begin{cases} 1/\alpha, & 0 \le x \le \alpha \\ 0, & \text{otherwise} \end{cases}$$

Obtain the maximum likelihood estimator of α. (4)

b) An unbiased die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. (3)

c) The joint distribution of X and Y is given by:

$f(x, y) = 4xy\ e^{-(x^2+y^2)};\ x \ge 0,\ y \ge 0$

Find the marginal distributions of X and Y. (3)

8. a) The mean of 5 observations is 15 and the variance is 9. If two more observations having values -3 and 10 are combined with these 5 observations, what will be the new mean and variance of the 7 observations? (4)

b) Let X be a random variable with p.m.f. given by the following table:

x	p(x)
-2	$\frac{3}{20}$
-1	$\frac{4}{20}$
0	$\frac{6}{20}$
1	$\frac{4}{20}$
2	$\frac{3}{20}$

Compute $E(X^2)$. (2)

c) A factory produces certain type of product by 3 machines. The respective daily production figures are:

Machine X: 3000 units

Machine Y: 2500 units

Machine Z: 4500 units

Past experience show that 1% of products produced by machine X, 1.2% by machine Y and 2% by machine Z are defective. A product is drawn at random. What is the probability that it has been produced by machine Y, if the drawn item is found to be defective. (4)

9. a) For a continuous distribution whose probability density function is given by:

$$F(x) = \frac{3x}{4}(2-x);\ 0 \le x \le 2;$$

find the expected value and variance of X. (4)

b) The following information about the two samples drawn from two normal populations is:

$$n_1 = 6,\ \sum (x_i - \bar{x})^2 = 60.2,\ n_2 = 8$$

and $\sum (y_i - \bar{y})^2 = 58.4.$

Construct 99% confidence interval for the ratio of population variance. [You may use $F_{(5,7)(0.005)} = 3.97,\ F_{(7,5)(0.005)} = 4.88$]. (6)

10. a) The following table gives the number of road accidents that occurred during the various days of the week: (5)

Day	No. of Accidents
Monday	14
Tuesday	15
Wednesday	8
Thursday	20
Friday	11
Saturday	9
Sunday	14

Test whether the accidents are uniformly distributed over the week at 1% level of significance.
[You may use $\chi^2_{0.01,\,6} = 16.81$, $\chi^2_{0.01,\,7} = 18.48$, $\chi^2_{0.01,\,8} = 20.09$].

b) A single letter is selected at random from the word 'STATISTICS'. What is the probability that it s a vowel? (2)

c) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with density function:

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \theta > 0 \text{ if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is unbiased estimator for θ. (3)