Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

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COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2023. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

$$f(x)=\frac{1}{(x+2)^3}, \forall x \in]-2, 2[$$

is continuous but not bounded in $]-2, 2[$. (4)

4. a) For the following sequences, find two subsequences which are convergent:

(i) $a_n = n[1+(-1)^n]$.

(ii) $a_n = \sin\left(\frac{n\pi}{3}\right)$. (3)

b) Check whether the following sequences $\{s_n\}$ are Cauchy, where

(i) $s_n = 1+2+3+...+n$

(ii) $s_n = \frac{4n^3+3n}{3n^3+n^2}$ (3)

c) Give an example of an infinite set with finite number of limit points, giving justification. (2)

d) Evaluate: $\lim_{x\to\infty}\left(\sqrt{2x^2+3x-2}-\sqrt{2x^2-3x+2}\right)$ (2)

5. a) Test for convergence the following series:

(i) $\sum_{n=1}^{\infty} \frac{7^n}{(3n+1)!}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\log n}}$ (4)

b) Test the following series for absolute and conditional convergence:

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{5}{3n+1}$

(ii) $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$ (4)

c) Show that the set $B = \{x \mid x^2 > 2\}$ is non-empty and bounded below. Is it bounded above? Justify. (2)

6. a) Show that the function f defined on **R** by

$$f(x)=\begin{cases} 3x^2\cos\frac{1}{2x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is derivable on **R** but f' is not continuous at x=0. (3)

10. a) Find the upper and lower integrals of the function f defined by

$$f(x) = \frac{7}{2} - 2x, \ \forall \ x \in [1,3].$$

Is f integrable over the interval $[1,3]$? Justify. (5)

b) Check whether the function $f(x) = \sin\frac{1}{x} (x \neq 0)$ is uniformly continuous in the interval $]0,2[$. Is it continuous? Justify. (3)

c) Find the value of $a \in \mathbf{R}$ for which

$$\lim_{x \to \infty} \frac{(3x+4)(x-1)(2x+1)}{ax^3 + x - 4} \text{ exists.} \quad (2)$$