MTE-08

# ASSIGNMENT BOOKLET

**Bachelor's Degree Programme**
**(B.Sc./B.A./B.Com.)**

# DIFFERENTIAL EQUATIONS

**(Valid from 1st January, 2023 to 31st December, 2023)**

**It is compulsory to submit the assignment before filling in the exam form.**

**School of Sciences**
**Indira Gandhi National Open University**
**Maidan Garhi**
**New Delhi-110068**
**(2023)**

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

**Instructions for Formatting Your Assignments**

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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**ROLL NO.:** ……………………………………………

**NAME:** ……………………………………………

**ADDRESS:** ……………………………………………

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**COURSE CODE:** …………………………….

**COURSE TITLE:** …………………………….

**ASSIGNMENT NO.:** ………………………….…

**STUDY CENTRE:** ……………………..….. **DATE:** ……………………….……………...

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

**We strongly suggest that you retain a copy of your answer sheets.**

We wish you good luck.

**ASSIGNMENT**

**(To be done after studying all the blocks)**

**Course Code: MTE-08**
**Assignment Code: MTE-08/TMA/2023**
**Maximum Marks: 100**

1. State whether the following statements are **true** or **false**. Justify your answer with the help of a short proof or a counter example: (5×2=10)

   a) If $\left(\frac{1}{y^4}\right)$ is the integrating factor of the differential equation

   $$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^2e^y - x^2y^2 - 3x)dy = 0,$$

   then its solution is $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$, where $c$ is a constant.

   b) Solution of the differential equation

   $$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 0,$$

   Satisfying the conditions $y(0) = y'(0) = y''(0) = 0$, and $y'''(0) = 1$, is $y = x - \sin x$.

   c) The homogeneous Pfaffian differential equation $z^3(x^2y - y^2z)dx + x^3(y^2z - z^2x)$ $dy + y^3(z^2x - x^2y)dz = 0$ is integrable.

   d) Equation $\sin(x+2y)p + \cos(2x-3y)q = z - \frac{1}{z}$ is linear.

   e) Equation $(1-y)u_{xx} + 2(1-x)u_{xy} + (1+y)u_{yy} + yu_x + xu_y = 0$ is hyperbolic outside the circle $(x-1)^2 + y^2 = 1$.

2. a) The rate at which the ice malts is proportional to the amount of ice at the instant. Find the amount of ice left after two hours if half of a quantity melts in 30 minutes. (4)

   b) Solve: $(D^2 - 6D + 13)y = 8e^{3x}\sin 4x + 3^x$. (4)

   c) The initial value problem $\frac{dy}{dx} = \frac{2}{x}y,\ y(0) = 0$ has two solutions $y = 0$ and $y = x^2$. Does this result violates existence and uniqueness theorem? Give reasons for your answer. (2)

3. a) Prove that the solutions of the differential equation $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ are given by $xy = c_1$ and $(x^2 - y^2) = c_2$, where $c_1$ and $c_2$ are constants. (4)

   b) By changing the dependent variable reduce the following equation to normal form and obtain its solution:

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x \, . \qquad (6)$$

4. a) Find the general integral of the equation

$$(3z^2 - 2yz - 2y^2)p + x(2y + z)q = x(y - 3z). \qquad (4)$$

b) Find the complete integral of the partial differential equation:

$$z = px + q + \frac{pq}{y}. \qquad (6)$$

5. a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions:

$u = 0$, when $x = 0$ and $x = \pi$,

$\frac{\partial u}{\partial t} = 0$, when $t = 0$; $u(x, 0) = x, 0 < x < \pi$. (6)

b) Solve:

$$\frac{dy}{dx} + \left(\frac{x}{1 - x^2}\right) y = x\sqrt{y}, \; y(0) = 1. \qquad (4)$$

6. a) Using the method of variation of parameters solve $\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$. (4)

b) What exactly is the advantage of transforming $f(x, y, z, p, q) = 0$, to $F\left(x, y, z, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = 0$? For the equation $z + 2u_3 - (u_1 + u_2)^2 = 0$, write down the auxiliary equations. (2)

c) Solve: $(x^2D^2 - y^2D^2 + xD - yD)\, z = 6xy^2$. (4)

7. A square plate $0 \le x \le 10, 0 \le y \le 10$, has the edges $x = 0, x = 10, y = 0$ maintained at zero degree temperature, while the temperature at the edge $y = 10$ is given by $20x - x^2$. Show that the temperature $\theta(x, y)$, satisfying Laplace's equation, is given by

$$\theta(x, y) = \sum_{m=1}^{\infty} F_m \sin\frac{m\pi x}{10}\left(e^{\frac{m\pi}{10}y} - e^{\frac{-m\pi}{10}y}\right),$$

where

$$F_m = \frac{1}{m\pi \sinh(m\pi)}\left[200(-1)^{m+1} + \frac{80}{m^2\pi^2}(1 - (-1)^m)\right]. \qquad (10)$$

8. a) Find the integral surface of the equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ passing through the line $x = 1, y = 0$. (5)

b) Find the complete integral of $(p + q)\,(px + qy) - 1 = 0$ by Charpit's method. (5)

9. a) Find the integral curves of the equations $\frac{dx}{x^2+y^2} = \frac{dy}{2xy} = \frac{dz}{(x+y)z}$. (5)

b) Solve: $(D^2 - D'^2 - 3D + 3D')\,z = e^{x+2y}$. (5)

10. a) By using method of symbolic operators find the general solution of
$(x^2D^2 - xD + 2)\,y = x\log x$. (5)

b) Solve the following differential equation by changing the independent variable,
$x^2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin x^2,\ x > 0.$ (5)