MTE-07

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

ADVANCED CALCULUS

Valid from 1st January 2023 to 31st December 2023

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….……………...

PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of the very thin variety) for writing your answers.
3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2023. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

ASSIGNMENT

Course Code: MTE-07
Assignment Code: MTE-07/TMA/2023
Maximum Marks: 100

1. State whether the following statements are true or false. Justify your answers. (10)

(a) The function $f(x,y)=\dfrac{(x+2)(y-2)}{x+y}$ is homogeneous on its domain.

(b) $\left\{3x+\dfrac{1}{2x}\middle|\, 0<x<1\right\}$ is bounded above.

(c) $f:\mathbf{R}^2\to\mathbf{R}$, $f(x,y)=\dfrac{1+x}{x}, x\neq 0$ is continuous on $[5,10]\times\mathbf{R}$.

(d) The function $f:\mathbf{R}^3\to\mathbf{R}$, $f(x,y,z)=e^{2\cos\pi}$ is integrable on the region bounded by the sphere, $x^2+y^2+z^2=1$.

(e) If $f:\mathbf{R}^2\to\mathbf{R}, f(x,y)=10$ and $D=[3,10]\times[-1,5]$, then $\iint_D f\,dx\,dy=420$.

2) (a) Show that the following limits do not exist: (6)

i) $\lim\limits_{(x,y)\to(0,0)}\dfrac{(x-y)^2}{x^2+y^2}$

ii) $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{|y|}$

(b) Show that $f(x,y)=4xy-x^2-y^4$ has a saddle point at $(0,0)$. (4)

3) (a) Find the directional derivation of

$$f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases} \qquad (5)$$

at $(0,0)$ in the direction $\left(\dfrac{\sqrt{3}}{2},\dfrac{1}{2}\right)$.

(b) Verify the chain rule for the Jacobians for the functions, (5)

$x=e^{3u}, y=2u+5v-w, z=u+v$
$u=p+6, v=q^3, w=3r.$

4. (a) Identify the intermediate forma and evaluate: (5)

i) $\lim\limits_{x \to \pi/2} \dfrac{\ln \sin x}{1 - \sin x}$

ii) $\lim\limits_{x \to \infty} 2x(\ln(x+1) - \ln x)$

(b) Show that

$$f(x, y) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ y \sin \dfrac{1}{y}, & y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$$

is continuous at $(0,0)$, but $f_x(0,0)$ does not exist. (5)

5. (a) Show that the equation $f(x, y) = y^2 - yx^2 - 2x^5 = 0$ defines a continuously differentiable function $y = g(x)$, in the neighbourhood of $(1, -1)$. Also find the derivative of g. (3)

(b) Show that the following functions are functionally dependent: (3)

$$u = 3x + 2y - z$$
$$v = x - 2y + z$$
$$w = x(x + 2y + z)$$

(c) Find the values of a and b, if

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$ (4)

6. (a) Evaluate the integral by converting to polar coordinates: (6)

$\iint\limits_D (4 - x^2 - y^2)\,dx\,dy$, where D is the region in the 1st quadrant and bounded by $x^2 + y^2 = 2x$.

(b) Find the work done by a force $\overline{F} = (x^2 y, xy^2)$ in moving a particle from $(0,0)$ to $(1,1)$ along $y = x^2$, and then from $(1,1)$ to $(2,1)$ along $y = 1$. (4)

7. (a) Find the surface area of the portion of the paraboloid $z = 4 - x^2 - y^2$, that lies above the xy-plane. (5)

(b) Show that the following integral is independent of path and evaluate it: (5)

$$\int_{(0,\pi)}^{(1,\pi/2)} (e^x \cos y \, dx - e^x \sin y \, dy)$$

8. (a) What are the domain and range of $f : \mathbf{R}^2 \to \mathbf{R}$ defined by $f(x, y) = \ln x + \dfrac{1}{y}$. (2)

(b) Evaluate f_{xy} at a point (x, y) for the function f defined by $f(x, y) = x \tan^{-1} y$. Using Schwarz's Theorem, evaluate f_{yx} at the point (x, y). (4)

(c) Let f be a function defined by

$$f(x, y) = \left(\frac{|x|}{1+|x|}, \frac{|y|}{1+|y|} \right)$$

Check whether the composition $g o f$ exists, where $g : \{(x, y) : x^2 + y^2 \leq 4\} \to \mathbf{R}$ is defined by $g(x, y) = xy$. Find $g o f$. (4)

9. (a) Find the volume below the plane $z = 1 - y$ and inside the cylinder $x^2 + y^2 = 1$, $0 \leq z \leq 1$. (5)

(b) Reverse the order of integration and integrate:

$$\int_0^2 \int_{y/2} (x + y)^2 dx\, dy.$$ (5)

10. (a) Find the second Taylor polynomial for $f(x, y) = e^{xy} \cos x$ about $(0, \pi/2)$. (5)

(b) If $f(x, y) = \begin{cases} x \sin \frac{1}{x} + y \sin \frac{1}{y}, & xy \neq 0 \\ 0 & , xy = 0, \end{cases}$ (5)