ASSIGNMENT BOOKLET

MTE-01

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

CALCULUS

(Valid from 1ˢᵗ January, 2023 to 31ˢᵗ December, 2023)

It is compulsory to submit the assignment before filling in the exam form.

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THE PEOPLE'S
UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: ………………………….

COURSE TITLE: ………………………….

ASSIGNMENT NO.: ………………………..…

STUDY CENTRE: ………………………...….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2023**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

(To be done after studying all the blocks)

1. Which of the following statements are **true** or **false**? Justify your answers: (10)

   i) The function f, defined by $f(x) = |x^2 - 3|$, is differentiable at all points in [1, 2].

   ii) $\lim\limits_{x \to 0} \dfrac{|\sin x|}{x} = 1$.

   iii) The curve $y = \dfrac{5 - 4x}{x^2 + 2}$ has no asymptote parallel to the y-axis.

   iv) The function f, defined by $f(x) = x + \cos x$, is monotonic in $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$.

   v) $\dfrac{d}{dx}\left[\displaystyle\int_0^{\sqrt{x}} \sin t^2 \, dt\right] = \cos x$.

2. a) State Lagrange's Mean Value Theorem. Can we use it to prove that, $0 < [ln(1+x)]^{-1} - x^{-1} < 1 \ \forall \ x > 0$? If yes, use the theorem to prove this inequality. Otherwise, prove the Intermediate Value Theorem. (6)

   b) Given a function f, defined by $f(x) = \sqrt{x^2 + 9}$, and given $L = 3, x_0 = 0, \varepsilon = 0.1$, find $\delta > 0$ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$. (4)

3. a) Differentiate $x^{\sin x} + (\sin x)^x$ with respect to x. (5)

   b) A curve is drawn to pass through the points given by the following table:

x	0.5	1	1.5	2	2.5	3	3.5	4
y	2.5	2	2.4	2.7	2.8	3	2.6	2.1

   Estimate the area bounded by the curve, the x-axis and the lines $x = 1, x = 4$, using Simpson's rule. (5)

4. Trace the curve: $a^3 x^2 = y^2(x + a)^3, a > 0$, clearly stating all the properties you used in the process. (10)

5. a) If $\cos^{-1}\left(\dfrac{y}{b}\right) = ln\left(\dfrac{x}{n}\right)^n$, check whether or not $x^2 y_{n+2} + (2n+1)x \, y_{n+1} + n^2 y_n = 0$. (5)

   b) Evaluate $\displaystyle\int \dfrac{dx}{(x-1)^2(x^2+4)}$. (5)

6. a) If $I_n = \int_0^{\pi/2} \cos^n x \sin nx \, dx$, $(n \geq 1)$, prove that $2I_{n+1} = I_n + \dfrac{1}{n+1}$. Hence evaluate I_2. (7)

   b) Find the domain of the function f, defined by $f(x) = \dfrac{1}{\sqrt{(x-3)(5-x)}}$. (3)

7. a) Find the length of the loop of the curve given by $3ay^2 = x(x-a)^2$, where a is a constant. (5)

   b) A company charges Rs. 55/- for a transistor set with a discount of Rs. 5/- for each set ordered in excess of 50. Find the largest size of order that will maximize the revenue of the company. (5)

8. a) Find the surface area of the solid formed by the rotation of the arc of the curve given by $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about the x-axis. (5)

   b) Evaluate $\int_0^2 \dfrac{(x-1)}{(x^2 - 4x + 5)} \, dx$. (5)

9. a) Given a function f, defined on $\mathbf{R}$, by:
   $$f(x) = \frac{x^2}{x^2 + 9},$$
   $l = 1$ and $\varepsilon = 0.1$, find $a > 0$ such that:
   $$x > a \Rightarrow |f(x) - l| < \varepsilon.$$ (3)

   b) Find $U(P, f)$ and $L(P, f)$, where $f(x) = x^2 - 4$ and $P = \{-2, -1, 0, 1, 2, 3\}$. (3)

   c) Evaluate: $\int_0^{\pi/2} \dfrac{dx}{1 - 4\cos^2 x}$. (4)

10. a) Find the equations of the tangent and the normal to the curve:
    $$x = e^t \cos t, \quad y = e^t \sin t$$
    at $t = 0$. (4)

    b) Find $\dfrac{dy}{dx}$, if:
    $$y = (\ln x)^{\cos x} + (\tan x)^{\csc x}.$$ (6)