## ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

## OPERATIONS RESEARCH

(Valid from $1^{\text {st }}$ January, 2023 to $31{ }^{\text {st }}$ December, 2023)

It is compulsory to submit the assignment before filling in the exam form.

THE PEOPLE'S UNIVERSITY
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(2023)

## Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, which would consist of one tutor-marked assignment for this course. The assignment is in this booklet.

## Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

## COURSE CODE:

COURSE TITLE:
ASSIGNMENT NO.: $\qquad$
STUDY CENTRE:
DATE: $\qquad$

## PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is valid only upto December, 2023. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.
7) It is compulsory to submit the assignment before filling in the exam form.

## We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

## ASSIGNMENT <br> (To be done after studying all the blocks)

Course Code: AOR-01
Assignment Code: AOR-01/TMA/2023
Maximum Marks: 100

1. Which of the following statements are true? Give reasons for your answer.
a) In an LPP if all the co-efficients of the objective function are positive and all the constraints are of ' $\leq$ ' type, the problem has a bounded optimal solution.
b) In any assignment problem, the optimal assignments are always along the main diagonal.
c) An ABC analysis is used to determine the re-order point in an inventory problem.
d) For a (M/M/1) queueing model $\lambda=2, \mu=3$, steady state solution exists.
e) An LPP with 4 variables and 3 constraints can have a maximum of 4 basic solutions.
2. a) A department has 5 jobs to be performed and there are five employees to perform these jobs. The time (in hours) each employee will take to perform each job is given in the following matrix:

|  |  | Employees |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | I |  |  |  |  |  | II | III | IV | V |
|  | Jobs | 10 | 5 | 13 | 15 | 16 |  |  |  |  |  |
|  | A | 10 | 18 | 13 | 6 |  |  |  |  |  |  |
|  | B | 3 | 9 | 18 | 2 | 2 |  |  |  |  |  |
|  | C | 10 | 7 | 2 | 7 | 12 |  |  |  |  |  |
|  | D | 7 | 11 | 9 | 7 | 12 |  |  |  |  |  |
|  | E | 7 | 9 | 10 | 4 |  |  |  |  |  |  |

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?
b) i) Formulate the dual of the LPP

Max. $5 \mathrm{x}_{1}+12 \mathrm{x}_{2}+10 \mathrm{x}_{3}$
Subject to $x_{1}+2 x_{2}+x_{3} \leq 10$
$3 x_{1}+x_{2}+5 x_{3} \leq 15$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0$
ii) Check whether $S_{1}=(0,35 / 9,20 / 9)$ is an optimal solution to the primal and $\mathrm{S}_{2}=(50 / 9,8 / 9)$, is an optimal solution to the dual without solving the problem.
3. a) In a factory, the machines break down at an average rate of 10 machines per hour. Assume that the number of machines is fairly large. The idle time cost of a machine is estimated to be Rs. 20 per hour. The factory works 8 hours a day. The factory manager is considering 2 mechanics for repairing the machines. The first
mechanic A takes about 5 minutes on an average to repair a machine and demands wages of Rs. 10 per hour. The second mechanic B takes about 4 minutes in repairing a machine and demands wages at the rate of Rs. 15 per hour. Assuming that the rate of machine breakdown is Poisson distributed and the repair rate is exponentially distributed, which of the two mechanics should be engaged?
b) Solve the following LPP by simplex method:

$$
\begin{align*}
& \text { Max } 9 \mathrm{x}_{1}+7 \mathrm{x}_{2}+7 \mathrm{x}_{3} \\
& \text { Subject to } \quad 3 \mathrm{x}_{1}+\mathrm{x}_{2}+2 \mathrm{x}_{3} \leq 12 \\
& \\
& \mathrm{x}_{1}+5 \mathrm{x}_{2}+3 \mathrm{x}_{3} \leq 30  \tag{5}\\
& \\
& \mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0 .
\end{align*}
$$

4. a) A fertilizer company distributes its products by trucks loaded at its only loading station. Both, company trucks and contractor's trucks are used for this purpose. It was found out that on an average every 5 minutes one truck arrived and the average loading time was 3 minutes. 40 percent of the trucks belong to the contractors. Making suitable assumptions, determine:
i) The probability that a truck has to wait.
ii) The waiting time of a truck that waits.
iii) The expected waiting time of contractor's trucks per day.
b) Find the shortest route from A to G using Bellman's Optimality Principle.

5. a) An insurance company has decided to modernize and refit one of its branch offices. Some of the existing office equipments will be disposed of but the remaining will be returned to the branch on completion of the renovation work. Tenders are invited from a number of selected contractors. The contractors will be responsible for all the activities in connection with the renovation work excepting the prior removal of the old equipment and its subsequent replacement.

The major elements of the project have been identified as follows along with their durations and immediately preceding element:

| Activity | Description | Duration <br> (weeks) | Immediate <br> Predecessor |
| :---: | :--- | :---: | :---: |
| A | Design new premises | 14 | - |
| B | Select the contractor and arrange details <br> with selected contractor | 7 | A |
| C | Arrange storage of equipment to be <br> retained | 5 | B |
| D | Arrange disposal of equipment not to be <br> retained | 2 | C |
| E | Procure new equipment | 4 | C, G |
| F | Renovations take place | 12 | C, D |
| G | Cleaning after the contractor has <br> finished | 2 | F |
| H | Bringing back retained equipment from <br> storage | 2 | G, E |

i) Draw the network diagram showing the inter-relations between the various activities of the project.
ii) Calculate the minimum time that the renovation can take from the design stage.
iii) Find the effect on the overall duration of the project if the estimates or tenders can be obtained in 2 weeks from the contractors by reducing their numbers.
iv) Calculate the 'independent float' that is associated with the non-critical activities in the network diagram.
b) The final table in Phase I of a Linear Programming Problem is given below:

| $\mathrm{C}_{\mathrm{B}}$ | Basic <br> Variables | 0 <br> $\mathrm{x}_{1}$ | 0 <br> $\mathrm{x}_{2}$ | 0 <br> $\mathrm{~S}_{1}$ | 0 <br> $\mathrm{~S}_{2}$ | -1 <br> $\mathrm{x}_{\mathrm{a}_{1}}$ | Sol. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{x}_{2}$ | 1 | 1 | 1 | 0 | 0 | 5 |
| -1 | $\mathrm{x}_{\mathrm{a}_{1}}$ | -1 | 0 | -1 | -1 | 1 | 3 |
|  |  | -1 | 0 | -1 | -1 | 0 | -3 |

Here $\mathrm{x}_{\mathrm{a}_{1}}$ is the artificial variable. Does this LPP have a feasible solution? Justify your answer.
6. a) ABC limited has 3 production shops supplying a product to 5 warehouses. Each shop has a specific production capacity and each warehouse has certain requirements. The costs of transportation, capacities and requirements are given below:

|  | I | II | III | IV | V | Supply |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Shop | A | 6 | 4 | 4 | 7 | 5 | 100 |
|  | B | 5 | 6 | 7 | 4 | 8 | 125 |
|  | C | 3 | 4 | 6 | 3 | 4 | 175 |
|  | Demand | 60 | 80 | 85 | 105 | 70 | 400 |

Find a basic feasible solution by Vogel's method and check its optimality.
b) Find the sequence that minimizes the total elapsed time required to complete the following tasks. Each job is processed in the order ABC.

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine A: | 12 | 6 | 5 | 11 | 5 | 7 | 6 |
| Machine B: | 7 | 8 | 9 | 4 | 7 | 8 | 3 |
| Machine C: | 3 | 4 | 1 | 5 | 2 | 3 | 4 |

7. a) A super bazaar in a city daily needs between 20 to 35 workers. The rush hours are between noon and 2 pm . The requirement of the workers at various hours is given in the following table:

| Time period <br> (in hours) | Number of workers <br> needed |
| :---: | :---: |
| $10-12$ | 22 |
| $12-1$ | 30 |
| $1-2$ | 35 |
| $2-3$ | 25 |
| $3-5$ | 20 |

The super bazaar can call up to 24 full time workers in a day. Others can be hired on contract for 4 hours at a stretch. The part-time workers can be employed beginning at $10 \mathrm{AM}, 12$ noon or 1 PM. Full time workers are given a lunch break of 1 hour, half of them taking their lunch from 12 noon to 1 PM and the other half between 1 PM to 2 PM. Full time workers are paid Rs. 90 per day and part-time workers are paid Rs. 40 for 4 hours.

Formulate the problem of finding the number of full time workers to be employed and the number of part-time workers to be called at $10 \mathrm{AM}, 12$ noon and 1 PM as a cost minimizing LPP.
b) In a factory a valve is consumed at the rate of 64 units per month. The inventory carrying costs are 20 paise per valve per month. It takes Rs. 250 to place and order and the lead time for receiving an order is assumed to be 0 . Determine:
i) EOQ
ii) Total cost per unit time
iii) Cycle time.
8. a) The details of a project consisting of 10 activities are given below:

| Activities | Duration (hrs.) |
| :---: | :---: |
| $1-2$ | 4 |
| $1-3$ | 12 |
| $1-4$ | 10 |
| $2-4$ | 8 |
| $2-5$ | 6 |
| $3-6$ | 8 |
| $4-6$ | 10 |
| $5-7$ | 10 |
| $6-7$ | 0 |
| $7-8$ | 10 |

Draw the network and find the project completion time.
b) Solve the following integer programming problem using branch and bound method:
Max: $\mathrm{z}=24 \mathrm{x}_{1}+17 \mathrm{x}_{2}$
s.t. $\quad 2 \mathrm{x}_{1}+\mathrm{x}_{2} \leq 12$
$20 x_{1}+15 x_{2} \leq 144$
$\mathrm{x}_{1} \leq 3, \mathrm{x}_{2} \geq 7$,
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$.
9. a) Use the dual simplex method to solve the following LPP:

Minimize: $z=10 y_{1}+8 y_{2}$
Subject to the constraints:

$$
\begin{aligned}
& 4 y_{1}+2 y_{2} \geq 5 \\
& 2 y_{1}+2 y_{2} \geq 3 \\
& y_{1}, y_{2} \geq 0 .
\end{aligned}
$$

b) Three custom officers separately check the luggages of the passengers at an airport. The passengers arrive $t$ an average rate of five per hour. The time a custom officer spends with a passenger is exponentially distributed, with mean service time 24 minutes. Find the probability that all the custom officers are idle. Also, find the probability that there are exactly 2 customers in the queue.
10. a) A factory manufacturers two articles A and B . To manufacture an article A , a certain machine has to work for 1.5 hours and in addition a craftsman has to work for 2 hours. To manufacture an article B, the machine has to work for 2.5 hours and in addition the craftsman has to work for 1.5 hours.

In a week the factory can avail 80 hours of machine time and 70 hours of craftsman's time. The profit on each article A is ₹ 5 and on each article B is ₹ 4. If all articles produced are sold away, find how many of each type of articles should be produced to earn the maximum profit per week using LPP.
b) i) Define $M / M / 1$ queuing model.
ii) A bank plans to open a single server desk in banking facility at a particular centre. It is estimated that on an average 28 customers re arriving each hour and it requires 2 minutes to process a customer's transaction. Determine:

1. The proportion of time that the system is idle.
2. Average time a customer will have to wait before reaching the server.
