M.Sc. ACTURIAL SCIENCE

Term-End Examination

June, 2011

MIA–009 F2F : GENERAL INSURANCE, LIFE AND HEALTH CONTINGENCIES

Time : 3 hours   *Maximum Marks : 100*

Note :   *In addition to this paper you should have available Acturial Tables and your own electronic calculators.*

SECTION - A

Answer *any five* questions.

1. (a)   A population is subject to constant force of   4
          mortality of 0.015.
          Calculate :
          (i)   The probability that a life aged 20
                 exact will die before age 21.25 exact.
          (ii)   The curtate expectation of a life aged
                 20 exact.

    (b)   Write down an alternative expression for   4
          each of the following statements. Use
          notation as set out in the" International
          Actuarial Notation" section of "Formulae
          and Tables for Examinations" when
          appropriate and express your answer as
          concisely as possible.

(i) Probability [maximum $\{T_x, T_y\} \leq n$]

(ii) $E[g(k_x)]$ where $g(k_x) = V^{k_x + 1}$ for $k_x < n$ and 0 for $k_x \geq n$

(iii) Probability $\{n < T_x \leq m\}$

(iv) $\lim\limits_{dt \to 0} \dfrac{\text{Probability [minimum}\{T_x, T_y\} \leq t + dt \mid T_x > t, T_y > t]}{dt}$

2. (a) In special mortality table with a select period   3
of one year, the following relationships are
true for all ages :

$0.5\ q_{[x]} = (0.33)\ q_x$

$0.5\ q_{[x] + 0.5} = (0.5)\ q_x$

Express $P_{[x]}$ in term of P_x

(b) For a certain group of pensioners,   5
$q_{75} = 0.05$ and $q_{76} = 0.06$. Calculate the
probability that a pensioner aged 75 exact
will die between ages 75.5 and 76.5
assuming :

(i) a uniform distribution of deaths
between consecutive birthdays.

(ii) a constant force of mortality between
consecutive birthdays.

3. Prove Thieles differential equation for an   8
endowment assurance issued to a life aged x to
be as follows.

$$\frac{\partial}{\partial t}\left[{}_t\overline{V}_{x:\overline{n}|} \right] = -\left(1 - {}_t\overline{V}_{x:\overline{n}|}\right) \mu_{x+t} + \delta\ {}_t\overline{V}_{x:\overline{n}|} + \overline{P}_{x:\overline{n}|}$$

4. On 1 January 1992 a life insurance company issued a number of 20-year pure endowment policies to a group of lives aged 40 exact. In each case, the sum assured was Rs. 75000 and premiums were payable annually in advance.

On 1 January 2006, 500 policies were still in force, During 2006, 3 policy holders died, and no policy lapsed for any other reason.
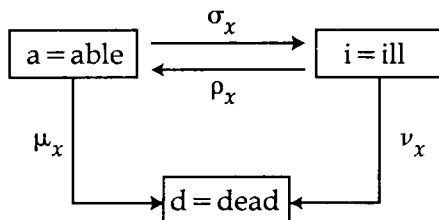
The office calculates net premiums and net premium reserves on the following basis :

Interest   :   4%per annum

Mortality :   AM 92 select

(a) Calculate the profit or loss from mortality   7
for this group for the year ending 31 December 2006.

(b) Explain why the mortality profit or loss has   1
arises.

5. Consider the following three - state illness-death model

Let $_tP_x^{jk}$ denotes the probability that a life in state j at age x will be in state k at age $x+t$, and let $_tP_x^{\overline{jj}}$ denotes the probability that a life in state j at age x will remain in state j for at least t years. Given a constant force of interest of δ *pa*, write down integral expressions for the expected present values of each of the following benefits.

(a) A benefit of Rs 50,000 payable immediately on the death of a life aged 50, provided that death occur within the next 10 - years.   **3**

(b) A benefit of Rs 50000 payable immediately on the death of a life aged 50, provided that death occurs within the next 10 years and the life has been sick for at least a year at the time of death.   **3**

(c) A sickness benefit of Rs 5000 *pa* payable continuously to a life aged 50 throughout any period of sickness. Benefit cease at age 60.   **2**

6. (a) (i)   In the context of with - profit policies, describe the super compound method of adding bonuses.   **2**

        (ii) Suggest a reason why a life insurance company might use the super compound method of adding bonuses as oppose to the compound method.   **2**

(b) A life insurance company sells an annual premium whole life assurance policy where the sum assured is payable at the end of the year of death. Expenses incurred at the start of each policy year, and claim expenses are nil.

(i) Write down a recursive relationship **3** between the gross premium provisions at successive durations, with provisions calculated on the premium basis. Define all the symbols that you use.

(ii) Explain in words the meaning of the **1** relationship.

7. A pension scheme provides a pension of $(n/60)$ ths **8** of final average salary payable at normal age retirement. However, n is limited to a maximum of 40 years. Write down a commutation function formula for calculating the total service liability for a member currently aged x, with a whole number of years' past service who joined the scheme :

(a) after age 25, and

(b) before age 25.

SECTION - B

Answer *any four* questions :

8. A life insurance company issues identical deferred annuities to each of 100 women aged 63 exact. The benefit is Rs 5000 per annum payable continuously from a woman's 65th birthday, if still alive at that time, and for life thereafter.

    (a) Write down for an expression for the random variable for the present value of future benefits for one policy at outset. **4**

    (b) Calculate the total expected present value at outset of these annuities **3**

    Basis :

    Mortality : PFA 92C20

    Interest : 4% per annum

    (c) Calculate the total variance of the present value at outset of these annuities, using the same basis as in part (ii). **8**

9. A life insurance company issues a 10-year decreasing term assurance to a man aged 50 exact. The death benefit is Rs 100000 in the first year, Rs 90000 in the 2nd year, and decreases by Rs 10000 each year so that the benefit in the 10th year is Rs 10000. The death benefit is payable at the end of the year of death.

    Level premiums are payable annually in advance for the term of the policy, ceasing on earlier death.

(a) Calculate the annual premium. 9

Basis :

Interest : 6% per annum

Mortality : AM92 Select

Initial expenses : Rs 200 and 25% of the total annual premium (all incurred on policy commencement)

Renewal expenses : 2% of each premium from the start of the 2nd policy year and Rs 50 per annum, inflating at 1.923% per annum, at the start of the second and subsequent policy years.

Claim expenses : Rs 200 inflating at 1.923% per annum.

Inflation : For renewal and claim expenses the amounts quoted are at outset, and the increases due to inflation start immediately.

(b) Write down an expression for the gross 3 future loss random variable at the end of the ninth year, using whatever elements of the basis in (i) that are relevant.

(c) Calculate the gross premium reserve at the 3 end of the ninth year, using the premium basis.

10. (a) A male life aged 63 years exact and his **10** spouse aged 60 exact buys a joint life annuity. The contract provides the following three benefits :

- Benefit A : an annuity certain of Rs. 20000 per annum payable monthly in advance during first 10 years of the contract.

- Benefit B : A deferred annuity of Rs. 20000 per annum payable monthly in advance. This benefit commences immediately after the 10 year expiry period of Benefit A. This benefit is payable as long as both spouse are alive on the due date of the annuity instalment.

- Benefit C : A deferred reversionary annuity of Rs 15000 per annum payable monthly in advance. The first annuity instalment is payable after one month following the later of the expiry of the 10 year benefit A period or the death of any one of the two annuitants.

  Calculate the single premium for this contract. You should use the following assumptions to calculate the premiums.

Mortality : PA92C20

Interest : 4% per annum

Expenses : Policy acquisition expenses of Rs 500 per contract.

Commission : 2% of the single premium.

(b) Prove the following result **5**

$$_nq^1_{xx} = \frac{1}{2}\,_nq_{xx}$$

11. A life insurance company issues a three year unit **15** linked endowment assurance contract to a male life aged 62 exact under which level annual premiums of Rs 10000 are payable in advance throughout the term of the contracts or until earlier death. 85% of each year's premium is invested in units at the offer price.

There is a bid offer spread in unit values, with the bid price being 95% of the offer price.

There is an annual management charges of 1.25% of the bid value of units. Management charges are deducted at the end of each year, before death or maturity benefits are paid.

On the death of the policy holder during the term of the policy, there is a benefit payable at the end of the year of death of Rs 20000, or the bid value of the units allocated to the policy, if greater. On maturity, 115% of the full bid value of the units is payable.

The company holds unit provisions equal to the full bid value of the units. It sets up non-unit provisions to zeroise any negative non-unit fund cash flows, other than those occurring in the first year.

The life insurance company uses the following assumptions in carrying out profit tests of this contract :

Mortality : AM92 Ultimate

Expenses : Initial Rs 600, Renewal Rs 100 at the start of the second and third policy years.

Unit fund growth rate : 8% per annum.

Non unit fund interest rate : 4% per annum.

Non unit fund provision basis : AM92 Ultimate mortality, interest 4% per annum.

Risk discount rate 15% per annum.

Calculate the profit margin on the contract.

12. An employer provides the following benefits for his employees.

- immediately on death in service, a lump sum of Rs 20000.

- immediately on withdrawal from service (other than on death or in ill health), a lump sum equal to Rs 1000 for each completed year of service

- On survival in service to age 65, a pension of Rs 2000 *pa* for each complete year of service, payable monthly in advance from age 65 for 5 years certain and life thereafter.

The independent rate of decrement for the employees are as follows.

Age x	q_x^d	q_x^i	q_x^w
62	0.018	0.10	0.020
63	0.020	0.15	0.015
64	0.023	0.20	0.010

Each decrement occurs uniformly over each year of age in its single decrement table.

(a) Construct a multiple decrement table with radix $(al)_{62} = 100000$ to show the number of deaths, ill-health retirements and withdrawals at ages 62,63 and 64, and the number remaining in employment until age 65.   6

(b) Calculate the expected present value of each of the above benefits for a new entrant aged exactly 62. Assume that interest is 6% *pa* before retirement and 4% *pa* thereafter, and that mortality after retirement follows the PMA92C20 table.   9

13. (a) Explain what is meant by the following   **9**
terms and give an example of each :

(i)   temporary initial selection.

(ii) time selection

(iii) spurious selection.

(b) The following data relate to a certain country   **6**
and its biggest province :

Age - group	Country		Province
	Population	Deaths	Population
0 - 19	2900000	580	800000
20 - 44	3500000	2450	1000000
45 - 69	2900000	20300	900000
70 and over	700000	49000	300000
Total	10,000,000	72330	3,000,000

The population figures are from a mid-year census along with the deaths that occurred in that year.

There were 25344 deaths in the province in total. Calculate the Area comparability factor and a standardised mortality rate for the province.